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COSMOLOGICAL INFLATION IN TENSOR-MULTI-SCALAR THEORY OF GRAVITATION

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In this paper, it is shown that the Tensor-Multi-Scalar Theory of Gravity (TMS TG) allows for inflationary solutions for the vacuum case, that is, in the absence of material source. For the two-field TMS TG, solutions are found in the presence of a source in the form of the scalar field in the slow-roll regime, when the gravitational part is given in the Einstein frame, and the action of the non-gravitational (material) field S_m is given in the Jordan frame. In this case, classes of power-law and de Sitter inflation solutions for various potentials of the non-gravitational scalar field are found.

Keywords: tensor-multi-scalar gravity, inflationary solutions.

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Introduction

The implementation of a scalar field in general relativity as a source of the gravitational field often meets with criticism on the basis of the fact that scalar fields are not detected experimentally. By now, a reasonable from the physical point of view argument, based on the discovery of the Higgs boson in the LHC experiment in 2000, has appeared. Thus, the scalar field describing the Higgs boson can be considered as the source of the gravitational field of the early Universe. Moreover, the Higgs field can be regarded as the inflaton leading to the early acceleration in the expansion of the universe (inflation) [1].

Turning to earlier ideas about the inclusion of the scalar field in gravitation, we give some historical remarks.

Jordan in 1949 [2] noted that in the Klein-Gordon theory of unification of gravity and electromagnetism, in assessing the scale of the 5th dimension, a new macroscopic interaction of gravitational stress, the carrier of which is a scalar field, inevitably arises. Since such a scalar field led to the estimation of the 5-th dimension through the coordinates of space-time, it subsequently became known as "compacton."

Fierz (1956) [3], Jordan (1959) [2], Brans and Dicke (1961) proposed the theory of gravity, described by the metric tensor and the scalar field with non-minimal coupling to gravity. Such a model contains only one free parameter, the tendency of which to infinity leads to the coincidence of the theory with general relativity. Later, Bergmann (1968) [5], Nordtvedt (1970) [6] and Wagoner (1970) [7] generalized the Fierz-Jordan-Brans-Dicke theory to the case more general scalar-tensor theory of gravitation due to the free function in front of the kinetic term and the introduction of the self-interaction potential for the scalar field.

Interest in the generalization of scalar-tensor theories of gravitation is caused, in particular, by the failure of the Fierz-Jordan-Brans-Dicke theory to give fundamentally new (different from general

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relativity) results of calculations for experiments in the solar system on the basis of post-Newtonian formalism. The tensor-multi-scalar theory of gravity (TMS TG), according to its authors, Damour and Esposito-Farese [8], provides reasonable predictions for four different observation regimes: 1) quasi-stationary weak fields regime (in the conditions of the solar system); 2) rapidly varying weak fields (gravitational waves); 3) quasi-stationary strong fields (neutron stars or black holes); 4) the effects of mixing strong fields and radiation for gravitational radiation in a system of many compact bodies. A detailed exposition of the above results can be found in the pioneering work of Damour and Esposito-Farese in 1992 [8]. It should be noted that TMS TG has much in common with the self-gravitating nonlinear sigma model [9] and the chiral cosmological model [10].

The tensor-multi-scalar theory of gravitation is a natural extension of the scalar-tensor theory. Its generalization to an arbitrary number of scalar fields, connected non-minimal with curvature, was proposed in the papers of 1992 [8] and 1995 [11]. Interest in this model appeared several years ago, after the statement that the Higgs field supports the early inflation, provided that the Higgs field is not minimally coupled to the gravity of [1], [12]. Also there are models in which dark matter [13] and relativistic stars [14] are described. In the paper [13], solutions are found in TMS TG for a homogeneous and isotropic universe in the Jordan frame by the dynamic system method in the two-field model. In the work [13] several scalar fields interacting with gravity are considered and dust solutions are found, as well as the solutions for the era of radiation dominance and matter dominance. By now, the inflationary model of the universe has become an integral part of the cosmological theory, since it solves the problems of flatness, horizon, the formation of the large-scale structure of the universe, and others. At the same time, the inflationary theory is reliably confirmed by the observational data of the observatories COBE, WMAP, PLANCK, BICEP2. All this suggests that along with the coordination of new (modified) gravitation theories in quasi-stationary weak gravitational fields in the solar system, gravitational waves, compact stellar objects, it is necessary to investigate the features of cosmological inflation to confirm the consistency of inheriting the progress of cosmological models based on general relativity . In this paper, we conduct a study of cosmological inflation in TMS TG with the inflaton for the case of two gravitational scalar fields.

It was proposed in [9] the method for constructing solutions for Chiral Cosmological Model (CMC) using the ansatz method. We are looking for solutions provided that the non-gravitational scalar field is in the slow-roll regime. Section 2 presents the general equations of TMS TG, the metric of the two-component CCM and, accordingly, the metric of the homogeneous and isotropic universe is chosen. Section 3 fixes the choice of non-gravitational matter as the self-interacting scalar field in the Jordan frame. General equations of cosmological dynamics are given in Section 4. It also presents a simplification of the model for the slow-roll regime and specifies the choice of special ansatzes that simplify the solution of the system of equations of cosmological dynamics for TMS TG. Section 5 is devoted to solutions for the first ansatz, a general algorithm for constructing solutions is indicated, and solutions are found for the power-law inflation and de Sitter inflation for various choices of self-interacting potentials of the scalar field. Section 6 presents similar studies for the second ansatz. The Conclusion summarizes the results and indicates the further directions of the research.

1. General Equations

Following the approach proposed by Damur and Esposito-Farese (1992) [8], we consider TMS TG in Einstein frame (without a non-minimal interaction of the scalar curvature to the scalar gravitational fields), when the action of the matter field as a source of gravitation is considered in the "physical" metric $\widetilde{g}_{\mu\nu}$, conformally related to the metric in the Einstein frame $g^*_{\mu\nu}$: $\widetilde{g}_{\mu\nu} = \Omega^2(\varphi)g^*_{\mu\nu}$. Thus, we consider the tensor-multi-scalar model of the theory of gravity with the action

$$S = \frac{1}{\kappa_*}\int d^4x\sqrt{-g_*}\left[\frac{R_*}{2} - \frac{1}{2}g_*^{\mu\nu}h_{AB}\varphi^A_{,\mu}\varphi^B_{,\nu} - W(\varphi^C)\right] + S_m[\psi_m, \Omega^2(\varphi)g^*_{\mu\nu}]. \tag{1}$$

To match our notation in the action (1) with the conventions in [8], [14], we can use the relations: $h_{AB} = 2\gamma_{AB},\ 2B(\varphi) = W(\varphi^C)$. Here and below (*) means that the analysis is carried out in the Einstein picture; $\kappa_* = \kappa$ is the Einstein gravitational constant, R_* is the scalar curvature, $g_* = det(g^*_{\mu\nu})$. To shorten the record, we use $\varphi_{,\mu} = \partial_\mu\varphi$. The Greek indices $\mu, \nu, ... = 0, 1, 2, 3$ define the coordinates of the space-time. The capital Latin indices $A, B, C, ... = 1, 2...N$ give N scalar fields. In what follows, the set of scalar fields $\{\varphi^1, \varphi^2, ...\varphi^N\}$ will be denoted by $\varphi := \{\varphi^1, \varphi^2...\varphi^N\}$.

We determine the Energy-Momentum Tensor (EMT) $T^*_{\mu\nu}$ of the non-gravitational field [8], taking into account that the matter is distributed in the space-time $\widetilde{g}_{\mu\nu} = \Omega^2(\varphi)g^*_\nu$

$$T^{(m)*}_{\mu\nu} = \frac{2}{\sqrt{-g_*}}\frac{\delta S_m[\psi_m, \Omega^2(\varphi)g^*_{\rho\sigma}]}{\delta g^{\mu\nu}_*}. \tag{2}$$

In this case, the energy conservation equation takes the form:

$$\nabla^\mu_* T^{(m)*}_{\mu\nu} = \frac{\partial \log\Omega(\varphi)}{\partial\varphi^B} T^{(m)*}\nabla^*_\nu\varphi^B, \tag{3}$$

where the trace of material EMT is defined by contraction with metric tensor $g^{\mu\nu}_*$: $T^{(m)*} = T^{(m)*}_{\mu\nu}g^{\mu\nu}_*$.

Varying the action (1) with respect to the metric $g^{\mu\nu}_*$, we obtain the equation of the gravitational field, written in terms of the EMT trace:

$$R^*_{\mu\nu} = h_{AB}\varphi^A_{,\mu}\varphi^B_{,\nu} + W(\varphi)g^*_{\mu\nu} + \kappa(T^{(m)*}_{\mu\nu} - \frac{1}{2}T^{(m)*}g^*_{\mu\nu}). \tag{4}$$

Field equations in general form are obtained by varying the action (1) with respect to the fields φ^A:

$$\Box_{*g}\varphi^A - \Gamma^A_{BC}(\varphi^D)g^{\mu\nu}_* \nabla^*_\mu \varphi^B \nabla^*_\nu \varphi^C - h^{AB}\frac{\partial W(\varphi)}{\partial\varphi^B} = -\kappa h^{AB}\frac{\partial \ln\Omega(\varphi)}{\partial\varphi^B}T^{(m)*}, \tag{5}$$

where $\Box^*_g = \nabla^*_\mu\nabla^\mu_*$.

The gravitational part of the action (1) in the absence of the second term S_m corresponds to the Chiral Cosmological Model (CCM) when choosing natural units, including $\kappa = 1$. Thus, the solutions obtained in a number of papers for the CCM [20–22], can be considered as vacuum solutions of TMS TG and confirm the existence of exact solutions of inflationary nature. As noted in the works [20–22], the consideration of two chiral fields interacting in a kinetic and potential way leads to results that can not be obtained for a single field.

In the present paper, we introduce the material component S_m as a scalar source in the standard Friedmann cosmological model with the self-interacting potential and consider the slow-roll regime.

The scalar component of the action (1) of the gravitational field is chosen in the representation of the two-component CCM with the metric of the target space:

$$d\sigma^2 = h_{11}d\phi^2 + h_{22}(\phi, \chi)d\chi^2. \tag{6}$$

Here we use the notation for the chiral fields: $\varphi^1 = \phi,\ \varphi^2 = \chi$.

The space-time metric of a homogeneous and isotropic universe will be written in the Friedman-Robertson-Walker representation (FRW)

$$ds^2_* = -dt^2_* + a^2_*(t)dl^2_* = -dt^2_* + a^2_*(t)\left(\frac{dr^2}{1-\epsilon r^2} + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2\right), \tag{7}$$

where $\epsilon = -1, +1.0$, which corresponds to an open, closed and spatially flat universe. Note that instead of considering an open and closed universe, we can remain in the spatially flat universe filled by the scalar field and the perfect fluid with the equation of state $p_{cur} = -3\rho_{cur},\ \rho_{cur} = -\epsilon/(3a^2)$ [23].

2. Choice of non-gravitational matter

In cosmological models of inflation, scalar fields with the self-interacting potential as a source of gravity are actively used. Therefore, we specify the action of matter following [8], as the action of the scalar field in the Jordan frame:

$$S_m = \int d^4x\sqrt{-\widetilde{g}}\left[-\frac{1}{2}\psi_{,\mu}\psi_{,\nu}\widetilde{g}^{\mu\nu} - \widetilde{V}(\psi)\right]. \tag{8}$$

As is known, there is a conformal connection between Einstein frame and the Jordan frame $\widetilde{ds}^2 = \Omega^2(\varphi)ds_*^2$. Hereinafter, $(\widetilde{\ })$ points to the description in the Jordan frame. Using (7), the following relations are established:

$$\widetilde{dt} = \Omega(\varphi)dt^*,$$
$$\widetilde{a}(t) = \Omega(\varphi)a^*(t).$$

The influence of matter (8) is transformed to the Einstein frame using the conformal transformation $\widetilde{g}_{\mu\nu} = \Omega^2(\varphi)g^*_{\mu\nu}$. Using (2), we obtain EMT:

$$T^{(m)*}_{\mu\nu} = \psi^*_{,\mu}\psi^*_{,\nu} - g^*_{\mu\nu}\left[\frac{1}{2}\psi^*_{,\alpha}\psi^*_{,\beta}g_*^{\alpha\beta} + V_*(\psi)\right]. \tag{9}$$

Varying (8) over the scalar field ψ, taking into account that $\widetilde{V}_*(\psi) = \Omega^{-4}(\varphi)V_*(\psi)$ and $\widetilde{\psi} = \Omega^{-1}\psi_*$, we arrive at the equation:

$$\Box_*\psi + V^*_{,\psi} = 0. \tag{10}$$

The trace of the EMT of non-gravitational matter takes the form: $T^{(m)*} = -(\psi^*_{,\mu}\psi_*^{;\mu} + 4V_*(\psi))$. Then the third term on the right-hand side of the equation (4) is transformed to the form:

$$T^{(m)*}_{\mu\nu} - \frac{1}{2}T^{(m)*}g^*_{\mu\nu} = g^*_{\mu\nu}V(\psi) + \psi^*_{,\mu}\psi^*_{,\nu}. \tag{11}$$

3. Cosmological dynamics equation

Equations (4, 5, 10) in the class of metrics (6,7) using the relation (11), are transformed to the form:

$$3H_*\dot{\chi}h_{22} + \partial_t(h_{22}\dot{\chi}) - \frac{1}{2}\frac{\partial h_{22}}{\partial\chi}\dot{\chi}^2 + \frac{\partial W(\phi,\chi)}{\partial\chi} = \kappa\frac{\partial\ln\Omega(\phi,\chi)}{\partial\chi}(\dot{\psi}_*^2 + 4V_*(\psi)), \tag{12}$$

$$\ddot{\phi}h_{11} + 3H_*\dot{\phi}h_{11} - \frac{1}{2}\frac{\partial h_{22}}{\partial\phi}\dot{\chi}^2 + \frac{\partial W(\phi,\chi)}{\partial\phi} = \kappa\frac{\partial\ln\Omega(\phi,\chi)}{\partial\phi}(\dot{\psi}_*^2 + 4V_*(\psi)), \tag{13}$$

$$H_*^2 = \frac{1}{3}\left[\frac{1}{2}h_{11}\dot{\phi}^2 + \frac{1}{2}h_{22}\dot{\chi}^2 + W(\phi,\chi)\right] + \frac{\kappa}{3}\left(\frac{1}{2}\dot{\psi}_*^2 + V_*(\psi)\right) - \frac{\epsilon}{a_*^2}, \tag{14}$$

$$\dot{H}_* = -\left[\frac{1}{2}h_{11}\dot{\phi}^2 + \frac{1}{2}h_{22}\dot{\chi}^2\right] - \frac{\kappa}{2}\dot{\psi_*}^2 + \frac{\epsilon}{a_*^2}, \tag{15}$$

$$\ddot{\psi} + 3H_*\dot{\psi} + V^*_{,\psi} = 0. \tag{16}$$

The system of equations (12 – 16) is the system of cosmological dynamic equations of the model under consideration. The consequence of equations (14 – 15) can be represented as the equations on the potential and kinetic constituents:

$$K(t) = \frac{1}{2}h_{11}\dot{\phi}^2 + \frac{1}{2}h_{22}(\phi,\chi)\dot{\chi}^2 + \frac{\kappa}{2}\dot{\psi_*}^2 = \frac{\epsilon}{a_*^2} - \dot{H}_*, \tag{17}$$

$$W(t) = \left[\dot{H}_* + 3H_*^2 + 2\frac{\epsilon}{a_*^2} - \kappa V_*(\psi)\right]. \tag{18}$$

Note, that the obtained equations (12, 13) differ by a non-zero right-hand side of the analogous equations (10.9 – 10.10) in the work [9]. Equations (14, 15) contain additional terms on the right-hand side that distinguishes them from the equations (10.11 - 10.12) of the work [9].

3.1 The slow-roll regime and the choice of the special ansatzes

For the analysis of theoretical predictions and their comparison with observational data, a slow-roll regime for the scalar field (16) is applied. The solution of the system of equations (12 - 16) will be sought using conditions for slow-roll: $|\dot{\psi}^2| \ll V(\psi)$ and $|\ddot{\psi}| \ll H|\dot{\psi}|$, discarding the second derivative $\ddot{\psi}$ and square of the first one $\dot{\psi}^2$. Then the system of equations (12 – 16) becomes:

$$3H_*\dot{\chi}h_{22} + \partial_t(h_{22}\dot{\chi}) - \frac{1}{2}\frac{\partial h_{22}}{\partial \chi}\dot{\chi}^2 + \frac{\partial W(\phi,\chi)}{\partial \chi} = 4\kappa \frac{\partial \ln \Omega(\phi,\chi)}{\partial \chi} V_*(\psi), \tag{19}$$

$$\ddot{\phi}h_{11} + 3H_*\dot{\phi}h_{11} - \frac{1}{2}\frac{\partial h_{22}}{\partial \phi}\dot{\chi}^2 + \frac{\partial W(\phi,\chi)}{\partial \phi} = 4\kappa \frac{\partial \ln \Omega(\phi,\chi)}{\partial \phi} V_*(\psi), \tag{20}$$

$$H_*^2 = \frac{1}{3}\left[\frac{1}{2}h_{11}\dot{\phi}^2 + \frac{1}{2}h_{22}\dot{\chi}^2 + W(\phi,\chi)\right] + \frac{\kappa}{3}V_*(\psi) - \frac{\epsilon}{a_*^2}, \tag{21}$$

$$\dot{H}_* = -\left[\frac{1}{2}h_{11}\dot{\phi}^2 + \frac{1}{2}h_{22}\dot{\chi}^2\right] + \frac{\epsilon}{a_*^2}, \tag{22}$$

$$3H_*\dot{\psi} + V^*_{,\psi} = 0. \tag{23}$$

Equations for the potential and kinetic parts (17, 18) are:

$$K(t) = \frac{1}{2}h_{11}\dot{\phi}^2 + \frac{1}{2}h_{22}(\phi,\chi)\dot{\chi}^2 = \frac{\epsilon}{a_*^2} - \dot{H}_*, \tag{24}$$

$$W(t) = \left[\dot{H}_* + 3H_*^2 + 2\frac{\epsilon}{a_*^2} - \kappa V_*(\psi)\right]. \tag{25}$$

The decomposition method (ansatz method) for finding solutions is described in the work [9]. For this system, we use the following two decomposition of the kinetic and potential components of the equations:

ANSATZ 1

$$h_{11} = const., \quad h_{11}\dot{\phi}^2 = -2\dot{H}_*, \tag{26}$$

$$h_{22}(\phi,\chi) = h_{22}(\chi), \quad h_{22}(\chi)\dot{\chi}^2 = 2\frac{\epsilon}{a_*^2}, \tag{27}$$

$$W(\phi,\chi) = W_1(\phi) + W_2(\phi) + W_3(\chi), \tag{28}$$

$$W_1(\phi(t)) = 3H_*^2 + \dot{H}_*, \tag{29}$$

$$W_2(\phi(t)) + W_3(\chi(t)) = 2\frac{\epsilon}{a_*^2} - \kappa V_*(\psi), \tag{30}$$

$$\chi = \sqrt{2}t. \tag{31}$$

It should be noted that the dependence of h_{22} on the field χ (27) in the metric of the target space (6) can be eliminated by the transformation $\hat{\chi} = \int \sqrt{h_{22}(\chi)}d\chi$. Nevertheless, the preservation of this dependence makes it possible to simplify integration of model equations and control the transition of the phantom zone when the sign of $h_{22}(\chi)$ is changing.

ANSATZ 2

$$h_{11} = const, \quad h_{11}\dot{\phi}^2 = -2\dot{H}_*, \tag{32}$$

$$h_{22}(\phi,\chi) = h_{22}(\phi), h_{22}(\phi)\dot{\chi}^2 = 2\frac{\epsilon}{a_*^2}, \tag{33}$$

$$W(\phi,\chi) = W_1(\phi) + W_2(\phi) + W_3(\chi), \tag{34}$$

$$W_1(\phi(t)) = 3H_*^2 + \dot{H}_*, \tag{35}$$

$$W_2(\phi(t)) + W_3(\chi(t)) = 2\frac{\epsilon}{a_*^2} - \kappa V_*(\psi), \tag{36}$$

$$\chi = \sqrt{2}t. \tag{37}$$

4. Algorithm for the solution for the 1st ansatz

The equation (20) can be represented as two equations taking into account the chosen decomposition (28), thus we obtain:

$$\ddot{\phi}h_{11} + 3H_*\dot{\phi}h_{11} + \frac{\partial W_1(\phi)}{\partial \phi} = 0, \tag{38}$$

$$\frac{\partial W_2(\phi)}{\partial \phi} = 4\kappa V_*(\psi)\frac{\partial \ln \Omega}{\partial \phi}. \tag{39}$$

Equation (38) is analogous to (10.27) in the work [9]. From equation (26) the chiral field $\phi(t)$ is defined in quadratures:

$$\phi(t) = \pm \int \sqrt{\frac{C-2}{h_{11}}\dot{H}_*}dt, \tag{40}$$

here $C = const$ is the constant of integration. It should be noted that the chiral field $\phi(t)$ is always real: the sign h_{11} defines the canonical ($h_{11} = +1$) or phantom field ($h_{11} = -1$). From the equation (29), knowing the dependence of H_* on t, one can find the potential of the field $W_1(\phi)$ using the transition $t \to \phi$ on the solution (40).

Assuming $\Omega(\phi,\chi) = \Omega_1(\phi)\Omega_2(\chi)$, it is obvious to find that $\ln\Omega(\phi,\chi) = \ln\Omega_1(\phi) + \ln\Omega_2(\chi)$. Then multiplying (39) by $\dot{\phi}$, we obtain:

$$\dot{W}_2(\phi) = 4\kappa V_*(\psi)\partial_t\left(\ln\Omega_1(\phi)\right). \tag{41}$$

To simplify the equation (27), we make the assumption that the chiral field $\chi(t)$ depends linearly on t:

$$\chi(t) = \sqrt{2}t, \tag{42}$$

then (27) is reduced to the form:

$$h_{22}(\chi) = \frac{\epsilon}{a_*^2}\Big|_{\chi=\sqrt{2}t}. \tag{43}$$

Let us consider in more detail the equation (19) in this decomposition:

$$3H_*\dot{\chi}h_{22} + \partial_t(h_{22}\dot{\chi}) - \frac{1}{2}\frac{\partial h_{22}}{\partial \chi}\dot{\chi}^2 + \frac{\partial W_3(\chi)}{\partial \chi} = 4\kappa V_*(\psi)\frac{\partial \ln\Omega(\phi,\chi)}{\partial \chi}. \tag{44}$$

We multiply (44) by $\dot{\chi}$ and perform the transition to the time dependence, taking into account the results of the section "10.2.1. Specificity of calculations " in the work [9]. As a result, we get

$$3H_*\dot{\chi}^2 h_{22} + \partial_t(h_{22}\dot{\chi})\dot{\chi} - \frac{1}{2}\dot{h_{22}}\dot{\chi}^2 + \dot{W}_3(t) = 4\kappa V_*(\psi)\alpha_\chi\dot{\chi}. \tag{45}$$

Make substitutions:

$h_{22}(\chi)\dot{\chi}^2 = 2\frac{\epsilon}{a_*^2}$,

$\dot{h_{22}}\dot{\chi}^2 = -4\epsilon\frac{H_*}{a_*^2} - 2h_{22}\ddot{\chi}\dot{\chi}$,

$\partial_t(h_{22}\dot{\chi})\dot{\chi} = -4\epsilon\frac{H_*}{a_*^2} - h_{22}\ddot{\chi}\dot{\chi}$

in equation (45). As a result, we find the time derivative $\dot{W}_3(t)$:

$$\dot{W}_3(t) = 4\kappa V_*(\psi)(\ln\dot{\Omega_2}(\chi)) - 4\epsilon\frac{H_*}{a_*^2}. \tag{46}$$

Consider the general picture of the solution, taking into account that in the slow-roll regime $V_*(\psi) \approx const$, $H_* \approx const.$. Then we can perform the integration over time in (41) and (46). As a result, we obtain:

$$W_2(\phi(t)) = 4\kappa V_*(\psi)\ln\Omega_1(\phi(t)), \tag{47}$$

$$W_3(\chi(t)) = 4\kappa V_*(\psi)(\ln\Omega_2(\chi(t))) + \frac{2\epsilon}{a_*^2}. \tag{48}$$

Thus the solution (47, 48) for $W_2(\phi(t))$ and $W_3(\chi(t))$ contains arbitrary functions of conformal transformation $\Omega_1(\phi(t))$ and $\Omega_2(\chi(t))$. In the framework of slow-roll conditions the relations $V_*(\psi) \approx const$, $H_* \approx const$ lead to the following: $\dot{\phi}^2(t) \approx 0$ from (26) and (40). That is, the field ϕ is constant. The situation of slow-roll regime for a material field leads to a similar situation for the first field $\phi(t)$, and hence the potential $W_1(\phi)$ will be constant.

On the basis of the analysis above, the algorithm for generating solutions is as follows. Define the inflationary expansion of the universe, choose the scale factor $a_*(t)$ (or, equivalently, the Hubble parameter $H_*(t)$) and find the inflation potential $V_*(\phi)$ as a function of time solving the equation (23). Using this data, we can determine the dependence of the first field on time from (40). The dependence of the potential $W_1(\phi(t))$ on time is determined from the equation (29), after which transition from the time dependence to the dependence on the field ϕ on the basis of the solution of the equation (40). The second component of the potential $W_2(\phi(t))$ is determined by integrating the equation (41). The remaining part of the potential $W_3(\chi)$ is determined by integrating the equation (46). Taking into account the slow-roll regime, the result of such integration is represented by the formulas (47, 48), which contain arbitrary functions of the conformal transition $\Omega_1(\phi)$ and $\Omega_2(\chi)$.

4.1 Power law inflation with given potential

In Friedmann cosmology the power law evolution of the scale factor has a special significance, since it describes the radiation stage and stage of the matter domination in the evolution of the universe. In inflationary models, the power law evolution of a scale factor allows us to find exact solutions for the background equations and to investigate in detail the equations for cosmological perturbations, calculation of the power spectrum and the spectral parameters (see, for example, [9]).

4.1.1 $a_*(t) = ct^m$, $V_*(\psi) = -D\ln\psi$

The choice of the logarithmic potential is related to its use in inflation models, for example, in the work [24].

The solution for the scalar field ψ is found from the equation (23) for the chosen dependences of the potential $V_*(\psi)$ and of the scale factor $a_*(t)$:

$$\psi(t) = t\sqrt{\frac{D}{3m}}. \tag{49}$$

Here $D, m = const$ and $m > 0$. In order to satisfy the slow-roll conditions for the field ψ, we require fulfillment of the condition $D \ll 3m$.

The Hubble parameter for power law evolution takes the form:

$$H_*(t) = \frac{m}{t}. \tag{50}$$

The dependence of the field $\phi(t)$ on time is found from the equation (40):

$$\phi(t) = \sqrt{2m}\ln t. \tag{51}$$

To find the potential of the field $W_1(\phi)$, substitute the value $H_*(t)$ from (50) into the equation (29) and then perform the transition from the time dependence to the dependence on the field $\phi(t)$. As a result, we get:

$$W_1(\phi) = m(3m-1)\exp\left(-\phi\sqrt{\frac{2}{m}}\right). \tag{52}$$

The solution for the potential $W_2(\phi)$ is determined from the equation (41) as follows. We substitute the dependence of non-gravitational field ψ on the time t to (41) and perform the integration. Then we restore the dependence on ϕ, using the solution (51). As a result, we obtain the expression for determining the potential $W_2(\phi)$ of the following form:

$$W_2(\phi) = -4D\kappa \left[\omega_1 \ln \sqrt{\frac{D}{3m}} + \frac{1}{\sqrt{2m}} \int \phi d\omega_1\right], \tag{53}$$

where $\omega_1 = \ln \Omega_1$.

Using (43) the component of the chiral metric $h_{22}(\chi)$ in the case of power law inflation becomes:

$$h_{22}(\chi) = \frac{\epsilon 2^m}{c^2 \chi^{2m}} \Big|_{\chi=\sqrt{2}t} . \tag{54}$$

Now we can find the functional dependence of the potential $W_3(\chi)$, substituting in the equation (46) the potential scalar field $V_*(\psi) = -D \ln \psi$, the scale factor $a_*(t) = ct^m$, the field χ (42) and the Hubble parameter (49). Then integrating the resulting equation, we perform the transition from t to χ: $t = \frac{\chi}{\sqrt{2}}$, finally get:

$$W_3(\phi) = -4D\kappa \left[\omega_2 \ln \sqrt{\frac{D}{6m}} + \int \ln \chi d\omega_2\right], \tag{55}$$

where $\omega_2 = \ln \Omega_2$.

Thus, the solution obtained is determined by the formulas (49 - 55) and does not use additional conditions on slow-roll approximation.

4.1.2 $a_*(t) = ct^m$, $V_*(\psi) = B\psi^k$.

The case of the monomial potential $V_*(\psi) = B\psi^k$ was considered in various inflationary models both for the massive scalar field $V(\phi) \propto \phi^2$, and in the case of a quantum ϕ^4 model.

The dependence of the field ψ on time is determined from the equation (23):

$$\psi = \left(Qt^2 + C_1\right)^{\frac{1}{2-k}}, \ k \neq 2, \tag{56}$$

where $Q = \frac{Bk(k-2)}{6m}$. In the following we set the constant C_1 equal to zero: $C_1 = 0$.

The case $k = 2$ leads to the solution

$$\psi = \psi_0 e^{-\frac{Bt^2}{3m}}, \ k = 2, \tag{57}$$

Here B, m, k, ψ_0 are constants. The scale factor does not changed, therefore $H_*, \phi(t), W_1(\phi)$ are also remain unchanged in accordance with (50-52).

The solution for the potential $W_2(\phi)$ is defined from the equation (41):

$$W_2(\phi) = 4\kappa B Q^{\frac{k}{2-k}} \int \exp\left[\frac{2k\phi}{\sqrt{2m(2-k)}}\right] d\omega_1, \ k \neq 2. \tag{58}$$

In the case $k = 2$ we obtain

$$W_2(\phi) = 4\kappa B\psi_0 \int \exp\left[-\frac{2B}{3m} \exp\left(\frac{2\phi}{\sqrt{2m}}\right)\right] d\omega_1, \ k = 2. \tag{59}$$

Here the function $\omega_1(\phi) = \ln \Omega_1(\phi)$.

The procedure for finding $W_3(\chi)$ is the same as for previous cases. The value of the chiral field $\chi(t)$ remains the same as in the formulas (42). The value of $h_{22}(\chi)$ will be:

$$h_{22}(\chi) = \frac{\epsilon 2^m}{c^2 t^{2m}} \Big|_{\chi=\sqrt{2}t} . \tag{60}$$

The functional dependence of the potential $W_3(t)$ takes the form

$$W_3(\chi) = 4\kappa B \left(\frac{Q}{2}\right)^{-\frac{k}{2-k}} \int \chi^{\frac{2k}{2-k}} d\omega_2 + \frac{2\epsilon}{a_*^2}, \ k \neq 2. \tag{61}$$

where the function $\omega_2(\chi) = \ln \Omega_2(\chi)$.

$$W_3(\chi) = 4\kappa B \int \exp\left(-\frac{B\chi^2}{3m}\right) d\omega_2 + \frac{2\epsilon}{a_*^2}, \; k = 2. \tag{62}$$

4.1.3 $a_*(t) = ct^m$, $V_*(\psi) = V_0 \exp(\mu\psi)$

The exponential potential in inflationary models leads to an exact solution within the framework of power law inflation. Consider this potential for our model.

The solution for the scalar field ψ is found from the equation (23) with the condition of the selected values of the potential $V_*(\psi)$ and scale factor $a_*(t)$:

$$\psi = \mu^{(-1)} \ln\left(\frac{6m}{t^2 V_0 \mu^2}\right). \tag{63}$$

Here $\mu, m, V_0 = const.$ The scale factor is unchanged, therefore $H_*, \phi(t), W_1(\phi)$ are unchanged in accordance with (50-52).

The solution for the potential $W_2(\phi)$ is defined from the equation (41):

$$W_2(\phi) = \frac{24\kappa m}{\mu^2} \int e^{-\sqrt{\frac{2}{m}}\phi} d\omega_1. \tag{64}$$

The condition on the field χ and on $h_{22}(\chi)$ remains the same (42, 43), respectively.

The functional dependence of the field potential $W_3(\chi)$ is determined by integration (46) and has the form:

$$W_3(\chi) = \frac{48\kappa m}{\mu^2} \int \chi^{-2} d\omega_2 + \frac{2\epsilon}{a_*^2}. \tag{65}$$

Thus, in this section we have obtained examples of exact solutions for power law inflation and for logarithmic, power law and exponential potentials.

4.2 De Sitter solution with given potential

De Sitter's solution in inflationary cosmology has an important application for the calculation of cosmological parameters and is associated with the approach of slow-roll. Considering the exponential evolution of the scale factor in the spatially flat Friedmann model of the universe with a scalar field, the exact solution says that the potential $V(\phi)$ and the field ϕ is constant. Let us consider the possible exact solutions in our model.

4.2.1 Features of the solution algorithm for $a_*(t) = a_0 \exp(H_0 t)$

The de Sitter solution $a_*(t) = a_0 \exp(H_0 t)$ means the constancy of the Hubble parameter $H_* = H_0 = const$. Then, from equation (22) taking into account the first equation from the system **ANSATZ 1**, we get that $\dot{\phi} = 0$. It means that the multiplication by $\dot{\phi}$, used in the derivation of the equation (41) and beyond, is not applicable in this case. Therefore, let us return to a special study of the original equations (19) - (23).

The equation (22) becomes

$$0 = -\frac{1}{2} h_{22}(\chi)\dot{\chi}^2 + \frac{\epsilon}{a_*^2}. \tag{66}$$

Obviously, two cases should be distinguished: $\epsilon = 0$ and $\epsilon \neq 0$.

Let us consider the first case when $\epsilon = 0$.

The solution of the system (19) - (23) is obtained directly and represents

$$\phi = const., \quad \chi = const., \quad \psi = const., \tag{67}$$

$$W_1 = 3H_0, \quad W_1 = const., \tag{68}$$

$$W_2 + W_3 = -\kappa V_*, \quad W_2 = const., \quad W_3 = const., \quad V_* = const.. \tag{69}$$

Thus, in the solution there are only relations to constants.

The case $\epsilon \neq 0$ requires more detailed description.

Assuming in equation (66), that $\epsilon \neq 0$ and $\chi = \sqrt{2}t$, we obtain

$$h_{22}(\chi) = \frac{\epsilon}{a_*^2} = \epsilon a_0^{-2} e^{-\sqrt{2}H_0\chi}. \tag{70}$$

Further, using this result, the equation (19) is reduced to the form

$$2\sqrt{2}H_0 h_{22}(\chi) + \frac{\partial W_3(\chi)}{\partial \chi} = 4\kappa \frac{\partial \omega_2}{\partial \chi} V_*(\psi(t)). \tag{71}$$

Multiplying the resulting equation by $\dot{\chi}$ and integrating with respect to time, we obtain

$$W_3(\chi) = 2\epsilon a_0^{-2} e^{-\sqrt{2}H_0\chi} + 4\kappa \int V_*(\psi(t))\dot{\omega_2} dt. \tag{72}$$

Thus, the equation for determining the dependence of the potential $W_3(\chi)$ may be obtained if the potential of non-gravitational field $V_*(\psi(t))$ is defined and the dependence $\omega = \omega_2(\psi(t))$ is known.

Taking into account the results obtained for the de Sitter space, the equation (20) is reduced to the form

$$\frac{\partial W_2(\phi)}{\partial \phi} = 4\kappa V_*(\psi(t)) \frac{\partial \omega_1(\phi)}{\partial \phi}. \tag{73}$$

An example of a solution of this equation is the case when

$$W_2 = W_2^{(0)} = const., \ \omega_1 = \omega_1^{(0)} = const.. \tag{74}$$

Using this solution, we consider the equation (21). Direct substitution of the obtained data into (21) leads to the equation

$$0 = W_2^{(0)} + 4\kappa \int V_*(\psi(t))\dot{\omega_2} dt + \kappa V_*(\psi(t)). \tag{75}$$

Differentiating the equation (75) in time, we arrive at the differential equation

$$4V_*\dot{\omega_2} + \dot{V_*} = 0, \tag{76}$$

from which by integration we can determine the relationship between a suitable conformal transformation and the potential of a non-gravitational field

$$\Omega_2(\phi(t)) = V_*(\psi(t))^{-1/4}. \tag{77}$$

The resulting relation allows us to write down the solution for the potential $W_3(\chi)$

$$W_3(\chi) = 2\epsilon a_0^{-2} e^{-\sqrt{2}H_0\chi} - W_2^{(0)} - \kappa V_*(\psi(t)), \tag{78}$$

which is true for any given potential of the non-gravitational field.

4.2.2 $a_*(t) = a_0 \exp(H_0 t)$, $V_*(\psi) = -D \ln \psi$

It is obvious that the Hubble parameter will be equal to some constant:

$$H_* = const. = H_0 > 0. \tag{79}$$

Solving the equation (23) in the slow-roll approximation, we obtain the following dependence of the field ψ on time:

$$\psi = \sqrt{\frac{2Dt}{3H_0}}. \tag{80}$$

Substituting the obtained in (80) dependence into the potential of the non-gravitational field

$$V_*(\psi) = -D \ln \psi, \tag{81}$$

and, restoring χ from t, we obtain

$$V_*(\chi) = -\frac{D}{2} \ln \left| \frac{\sqrt{2} D \chi}{3H_0} \right|. \tag{82}$$

Further, we define $W_3(\chi)$ from equation (78)

$$W_3(\chi) = 2\epsilon a_0^{-2} e^{-\sqrt{2} H_0 \chi} - W_2^{(0)} + \kappa \frac{D}{2} \ln \left| \frac{\sqrt{2} D \chi}{3H_0} \right|. \tag{83}$$

Extracting the dependence on χ, we represent the solution in the following form

$$W_3(\chi) = 2\epsilon a_0^{-2} e^{-\sqrt{2} H_0 \chi} + \kappa \frac{D}{2} \ln |\chi| + C_3, \tag{84}$$

where $C_3 = -W_2^{(0)} + \kappa \frac{D}{2} \ln \left| \frac{\sqrt{2} D}{3H_0} \right|$.

Thus, the solution is defined by formulae (68,70,74,77,80,82,84).

4.2.3 $a_*(t) = a_0 \exp(H_0 t)$, $V_*(\psi) = V_0 \exp(\mu\psi)$

Solving equation (23) in the slow-roll approximation, we obtain the dependence of the field ψ on t:

$$\psi = \frac{1}{\mu} \ln \left(\frac{3H_0}{V_0 \mu^2 t} \right). \tag{85}$$

Substituting obtained result in (85) solution for ψ in the definition of the potential

$$V_*(\psi) = V_0 \exp(\mu\psi), \tag{86}$$

we obtain

$$V_*(\psi) = \frac{3H_0\sqrt{2}}{\mu^2 \chi}. \tag{87}$$

Similarly to the previous case, we find the potential $W_3(\chi)$:

$$W_3(\chi) = 2\epsilon a_0^{-2} e^{-\sqrt{2} H_0 \chi} - W_2^{(0)} + \kappa \frac{3H_0\sqrt{2}}{\mu^2 \chi}. \tag{88}$$

Thus, the solution is defined by formulae (68, 70, 74, 77, 85, 87) and (88).

4.2.4 $a_*(t) = a_0 \exp(H_0 t)$, $V_*(\psi) = B\psi^k$

Solving equation (23) in slow-roll regime, we obtain the following dependence of the field ψ on time t:

$$\psi = (Ut)^{\frac{1}{2-k}}, \ k \neq 2, \tag{89}$$

where $U = \frac{Bk(k-2)}{3H_0}$.

The case when $k = 2$ leads to the following dependence ψ on time t

$$\psi = \exp\left(-\frac{2B}{3H_0} t \right), \ k = 2, \tag{90}$$

Let us consider the solution (89) in the case $k \neq 2$. Substituting the solution (89) into given potential

$$V_*(\psi) = B\psi^k, \tag{91}$$

we obtain

$$V_*(\chi) = B\left(U\chi/\sqrt{2}\right)^{\frac{k}{2-k}}. \tag{92}$$

In the standard way, we find the potential $W_3(\chi)$ from the solution (78)

$$W_3(\chi) = 2\epsilon a_0^{-2} e^{-\sqrt{2}H_0\chi} - W_2^{(0)} + \kappa B\left(U\chi/\sqrt{2}\right)^{\frac{k}{2-k}}. \tag{93}$$

The solution for this case is defined by formulae (68,70,74,77,89,92,93).

The case $k = 2$ leads to the following solution:

$$\psi = \exp\left(-\frac{2B\chi}{3\sqrt{2}H_0}\right), \tag{94}$$

$$V_*(\chi) = B\exp\left(-\frac{4B\chi}{3\sqrt{2}H_0}\right), \tag{95}$$

$$W_3(\chi) = 2\epsilon a_0^{-2} e^{-\sqrt{2}H_0\chi} - W_2^{(0)} + \kappa B\exp\left(-\frac{4B\chi}{3\sqrt{2}H_0}\right). \tag{96}$$

5. Algorithm for the solution for the 2nd ansatz

To simplify the equation for the connection of the components of the chiral metric $h_{22}(\phi)$ to (33), we assume that the chiral field χ depends linearly on t (37), then

$$h_{22}(\phi(t)) = \frac{\epsilon}{a_*^2(t)}. \tag{97}$$

We represent the equation (20) as two equations taking into account the chosen decomposition (32):

$$\ddot{\phi}h_{11} + 3H_*\dot{\phi}h_{11} + \frac{\partial W_1(\phi)}{\partial\phi} = 0, \tag{98}$$

$$-\frac{1}{2}\frac{\partial h_{22}(\phi)}{\partial\phi}\dot{\chi}^2 + \frac{\partial W_2(\phi)}{\partial\phi} = 4\kappa V^*(\psi)\frac{\partial\ln\Omega}{\partial\phi}. \tag{99}$$

The equation (98) similarly to (10.27) of the work [9] and to the equation (38). From equation (32) the chiral field ϕ is defined in quadratures by formula (40).

We assume that $\Omega(\phi,\chi) = \Omega_1(\phi)\Omega_2(\chi)$, and, obviously : $\ln\Omega(\phi,\chi) = \ln\Omega_1(\phi) + \ln\Omega_2(\chi)$. Then multiplying (99) by $\dot{\phi}$, and taking into account the relations (33) and (37), we obtain

$$\dot{W}_2(\phi) = 4\kappa V_*(\psi)\dot{\omega}_1 - 2\frac{\epsilon\dot{a}_*}{a_*^3}, \tag{100}$$

where $\omega_1 = \ln\Omega_1$. The resulting equation is similar to the equation on $\dot{W}_2(\phi)$ (47). The difference is the right side, containing a scale factor.

Let us consider in more detail the equation (19) in this decomposition:

$$3H_*\dot{\chi}h_{22} + \partial_t(h_{22}\dot{\chi}) + \frac{\partial W_3(\chi)}{\partial\chi} = 4\kappa V_*(\psi)\frac{\partial\ln\Omega_2(\chi)}{\partial\chi}. \tag{101}$$

We make the transition to the time dependence, taking into account the results of the section "10.2.1. Specificity of calculations" in work [9]. To this end, we multiply the equation (101) by $\dot{\chi}$ and obtain

$$3H_*\dot{\chi}^2h_{22} + \partial_t(h_{22}\dot{\chi})\dot{\chi} + \dot{W}_3(t) = 4\kappa V_*(\psi)\partial_t(\ln\Omega_2(\chi)). \tag{102}$$

From this equation we obtain $\dot{W}_3(t)$, substituting h_{22} from (97) and χ from (37). The result is

$$\dot{W}_3(t) = 4\kappa V_*(\psi)\partial_t(\ln\Omega_2(\chi)) - 2H_*\frac{\epsilon}{a_*^2}. \tag{103}$$

The algorithm for generating solutions is similar to that used for the system of equations **ANSATZ 1**. Specify the scale factor $a(t)$, which is responsible for the inflation solution, and the potential of the scalar field $V(\psi)$. Using this data, one can find functional dependences for the potentials of chiral fields $W_1(\phi)$ from (98), $W_2(\phi)$ from (100) and $W_3(\chi)$ from (103). In this way, we investigated the solutions for the power law and de Sitter inflation for the potentials of the scalar field by the following types: $V_*(\psi) = -D \ln \psi$, $V_*(\psi) = B\psi^k$, $V_*(\psi) = V_0 \exp(\mu\psi)$.

5.1 Power law inflation for given potentials

When we are considering power law inflation, some solution formulae remain the same as for the system of equations **ANSATZ 1**. Namely, $\phi(t)$ and $W_1(\phi)$ are represented by the formulae (51) and (52) respectively. Let us reproduce these formulae

$$W_1(\phi) = m(3m-1) \exp\left(-\phi\sqrt{\frac{2}{m}}\right), \tag{104}$$

$$\phi(t) = \sqrt{2m} \ln t. \tag{105}$$

We find the form of the chiral metric component for power law inflation using the equation (97):

$$h_{22}(\phi) = 2\frac{\epsilon}{c^2} \exp\left(-\sqrt{2m}\phi\right). \tag{106}$$

The solutions for the potentials $W_2(\phi)$ (100), $W_3(\chi)$ (103) are different from those obtained for the system of equations **ANSATZ 1**, and below they are listed in the table.

Potential	**Solution**
$V_*(\psi) = -D \ln \psi$	$\psi = t\sqrt{\frac{D}{3m}}$, $W_2(\phi) = -4D\kappa \int \ln\left(\sqrt{\frac{D}{3m}} \exp\left(\frac{\phi}{\sqrt{2m}}\right)\right) \frac{d\omega 1}{d\phi} d\phi + \frac{\epsilon}{c^2} \exp\left(-\frac{2\phi}{\sqrt{2m}}\right)$ $W_3(\chi) = -4D\kappa \int \ln\left(\sqrt{\frac{D}{3m}}\sqrt{2}\chi\right) \frac{d\omega 2}{d\chi} d\chi + \frac{\epsilon}{c^2 2^m \chi^{2m}}$
$V_*(\psi) = V_0 \exp(\mu\psi)$	$\psi = \mu^{-1} \ln\left(\frac{6m}{t^2 V_0 \mu^2}\right)$ $W_2(\phi) = 4\kappa \int \frac{6m}{\mu^2} \exp\left(-\frac{2\phi}{\sqrt{2m}}\right) \frac{\partial\omega 1}{\partial\phi} d\phi + \frac{\epsilon}{c^2} \exp\left(-\frac{2\phi}{\sqrt{2m}}\right)$ $W_3(\chi) = 4\kappa \int \frac{6m}{\mu^2} \frac{2}{\chi^2} \frac{\partial\omega 2}{\partial\chi} d\chi + \frac{\epsilon}{c^2 2^m \chi^{2m}}$
$V_*(\psi) = B\psi^k, k \neq 2$	$\psi = \left(-\frac{t^2(2-k)Bk}{6m}\right)^{\frac{1}{2-k}}$ $W_2(\phi) = 4\kappa B Q^{\frac{k}{2-k}} \int \exp\left[\frac{2k\phi}{\sqrt{2m}(2-k)}\right] \frac{\partial\omega 1}{\partial\phi} d\phi + \frac{\epsilon}{c^2} \exp\left(-\frac{2\phi}{\sqrt{2m}}\right)$ $W_3(\chi) = 4\kappa B Q^{\frac{k}{2-k}} \int \left(\frac{\chi}{\sqrt{2}}\right)^{\frac{2k}{2-k}} \frac{\partial\omega 2}{\partial\chi} d\chi + \frac{\epsilon}{c^2 2^m \chi^{2m}},\ Q = \left(\frac{Bk(2-k)}{6m}\right).$
$V_*(\psi) = B\psi^k,\ k = 2$	$\psi = \psi_0 e^{-\frac{Bt^2}{3m}}$ $W_2(\phi) = 4\kappa B\psi_0 \int \exp\left[-\frac{2B}{3m} \exp\left(\frac{2\phi}{\sqrt{2m}}\right)\right] \frac{\partial\omega 1}{\partial\phi} d\phi + \frac{\epsilon}{c^2} \exp\left(-\frac{2\phi}{\sqrt{2m}}\right)$ $W_3(\chi) = 4\kappa B\psi_0 \int \exp\left(-\frac{B\chi^2}{3m}\right) \frac{\partial\omega 2}{\partial\chi} d\chi + \frac{\epsilon}{c^2 2^m \chi^{2m}}$

5.2 De Sitter's inflation for given potentials

In this case, the class of solutions is substantially narrowed, since the analysis of the equations of the system **ANSATZ 2** leads to the only possibility with respect to the parameter ϵ, namely $\epsilon = 0$. Thus, for any given potential of non-gravitational scalar field $V_*(\psi(t))$, we have:

$$a_*(t) = a_0 \exp(H_0 t), \quad H_* = H_0 = const., \tag{107}$$

$$\phi = \phi_0 = const., \quad h_{22}(\phi_0) = 0, \tag{108}$$

$$W_1 = 3H_0, \quad W_2 = 0, \quad W_3 = -\kappa V_*(\psi), \quad V_* = V_0 \exp(-4\omega_2(\chi)). \tag{109}$$

Non-gravitational scalar field ψ is defined from equation

$$3H_0\dot{\psi}^2 + \dot{V}_* = 0. \tag{110}$$

Conclusion

The tensor-multi-scalar theory of gravitation, as a generalization of the scalar-tensor theory, can lead to a more accurate description of cosmological inflation, based on the nonminimal interaction of the Higgs field with gravity. Moreover, it is not excluded possibility that gravitational scalar fields of different origin can naturally explain the effect of accelerated expansion of the universe at present. Thus, the study of TMS TG can lead to a cosmological model, where naturally there are two stages of the accelerated expansion of the universe: early and late inflation. In this paper we studied exactly the early inflation.

The first conclusion, which we noted, is the fact that the solutions obtained earlier in the works of [20–22], confirm the existence of inflation solutions in TMS TG in the absence of a source of gravity, that is for the vacuum situation. Next, we consider the standard inflation, when the source of the gravitational field is the inflaton and the equations of cosmological dynamics are considered in the slow roll approximation. For this case, we found solutions for exponential inflation and de Sitter expansion. In doing so, we used the freedom of conformal transformation under the absence of non-minimal interaction in the Jordan frame.

The results obtained require further study in the light of the development of early inflation, namely, the change in the number e-folds in the transition to TMS TG and changes in the calculation of spectral parameters. This issue is planned to be investigated further and it will be represented in a separate publication.

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