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АНАЛИТИЧЕСКОЕ ОПИСАНИЕ ПОТОКА ПРОТОНОВ КОЛЬЦЕВОГО ТОКА ЗЕМЛИ ДЛЯ ПИТЧ-УГЛА 90 ГРАДУСОВСмолин С. В.^{а,1}^а Сибирский федеральный университет (СФУ), г. Красноярск, 660041, Россия

Как математическая модель предложено обыкновенное дифференциальное уравнение для аналитического описания перпендикулярного дифференциального потока заряженных частиц в магнитосфере Земли, которое зависит от времени t и нескольких параметров (параметр МакИлвейна, магнитное местное время или геомагнитная восточная долгота, индекс геомагнитной активности, показатель питч-углового распределения заряженных частиц или индекс анизотропии питч-углового распределения, но взятый для питч-угла 90 градусов при $t = 0$, средний показатель питч-углового распределения заряженных частиц на интервале времени вычисления, время жизни вследствие взаимодействий волна-частица). При определенных геофизических условиях и на временном интервале приблизительно не более чем три часа (когда индекс геомагнитной активности $Kp = \text{const}$) или на большем временном интервале, когда $Kp \approx \text{const}$, уравнение решается аналитически. Получено простое аналитическое решение, которое зависит от времени и нескольких параметров. Произведено сравнение результатов по предложенной модели и по полной модели для области питч-углов от 0 до 180 градусов. Получено для перпендикулярного дифференциального потока протонов кольцевого тока Земли очень хорошее согласие с максимальной относительной ошибкой приблизительно несколько процентов (для рассмотренного примера 3,23 процента). Экспериментальные данные были собраны спутником "Cluster". Модель позволяет также оценивать для разных геофизических условий время жизни вследствие взаимодействий волна-частица. Вывод уравнений представлен в аппендиксе.

Ключевые слова: магнитосфера, уравнение питч-угловой диффузии, перпендикулярный дифференциальный поток, взаимодействия волна-частица, новая модель.

THE ANALYTICAL DESCRIPTION OF THE EARTH'S RING CURRENT PROTON FLUX FOR THE PITCH ANGLE OF 90 DEGREESSmolin S. V.^{а,1}^а Siberian Federal University (SFU), Krasnoyarsk, 660041, Russia

As mathematical model is offered the ordinary differential equation for the analytical description of a perpendicular differential flux of the charged particles in the Earth's magnetosphere which depends on time t and several parameters (the McIlwain parameter, the magnetic local time or geomagnetic eastern longitude, the geomagnetic activity index, parameter of the charged particle pitch angle distribution or the pitch angle distribution anisotropy index but is taken for the pitch angle of 90 degrees at $t = 0$, the average parameter of the charged particle pitch angle distribution on an interval of time of calculation, the lifetime due to wave-particle interactions). Under the certain geophysical conditions and on a time interval approximately no more than three hours (when a geomagnetic activity index $Kp = \text{const}$) or on a greater time interval, when $Kp \approx \text{const}$, the equation is solved analytically. The simple analytical solution is received which depends on time and several parameters. Comparison of results on the offered model and on full model for the pitch angle range from 0 up to 180 degrees is lead. For a perpendicular differential flux of the Earth's ring current protons very good consent with the maximal relative error approximately some percent (for the considered example 3.23 percent) is received. Experimental data have been collected by the Cluster satellite. The model allows to estimate also for different geophysical conditions a lifetime due to wave-particle interactions. A conclusion of the equations is presented in an appendix.

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Keywords: magnetosphere, pitch angle diffusion equation, perpendicular differential flux, wave-particle interactions, new model.

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Introduction

The terrestrial ring current, which is an electric current flowing westward around the Earth, basically locates at distances between ~ 2 and $9 R_E$ (R_E is the average radius of the Earth). Its origin, growth and decay are strongly associated with geomagnetic storms. The trapped energetic (\sim tens of keV) positive ions (H^+ , He^+ and O^+) undergo an azimuthal drift and comprise the storm ring current. Three primary processes are found to be responsible for ring current decay: Coulomb collision processes and charge exchange, together with pitch angle diffusion by electromagnetic ion cyclotron waves [1].

Recently, numerous studies have performed on stormtime dynamics of radiation belt energetic particles and also for particles of the Earth's ring current since their fluxes are found to vary strongly. Such strong variation of dynamics of radiation belts and the Earth's ring current is considered as the contribution from wave-particle interactions, and from drift resonance associated with enhanced ultra low frequency waves. However simultaneous observation and corresponding modeling of pitch angle scattering of ring current protons by electromagnetic ion cyclotron waves during a nonstorm period ($Kp < 3$) have seldom been reported [2]. That is the primary purpose of this study.

1. The mathematical model

The 1D modified Fokker-Planck equation (or pitch angle diffusion equation) for the phase space density, describing "pure" pitch angle diffusion only with wave-particle interactions losses, can be expressed by [3, 4]

$$\frac{\partial f}{\partial t} = \frac{1}{\sin \alpha} \frac{\partial}{\partial \alpha} \left(\sin \alpha \cdot D_{\alpha\alpha} \frac{\partial f}{\partial \alpha} - \frac{\sin^2 \alpha \cos \alpha}{2L} \frac{dL}{dt} f \right) - \frac{f}{T_{wp}} + S_{\perp} \sin^2 \alpha \cdot f. \quad (1)$$

Here, f is the phase space density (referred to as distribution function), t is the time, L is the McIlwain parameter, α is the local pitch angle from 0 up to 180 degrees, $D_{\alpha\alpha}$ is the pitch angle diffusion coefficient [3, 4], dL/dt is the radial rate [3, 4], T_{wp} is the lifetime due to wave-particle interactions [3, 4], S_{\perp} is the perpendicular coefficient of the particle source function ($\alpha = 90^\circ$) [3, 4].

The equation (1) is original model in space of velocities which contains some mathematical models of pitch angle diffusion as particular cases, e.g., [5–7].

Using (1), as new mathematical model is offered the ordinary differential equation for the analytical description of a perpendicular differential flux of the charged particles ($\alpha = 90^\circ$) in the Earth's magnetosphere which depends on time and several parameters:

$$\frac{dj_{\perp}}{dt} + \left(-\frac{1}{2L} \frac{dL}{dt} + \frac{\gamma_{\perp} - \gamma_{\perp 0}}{\gamma_{\perp 0}(\gamma_{\perp 0} + 2) T_{wp}} \right) j_{\perp} = 0, \quad (2)$$

where j_{\perp} is the perpendicular differential flux of the charged particles ($j = 2mEf$, m is the charged particle mass, E is the particle energy), $\gamma_{\perp 0}$ is the well-known (when $j = j_{\perp} \sin^{\gamma} \alpha$) parameter of the charged particle pitch angle distribution (or the pitch angle distribution anisotropy index) but is taken for the 90° pitch angle at $t = 0$, γ_{\perp} is the average parameter of the charged particle pitch angle distribution on an interval of time of calculation (it is supposed, that $\gamma_{\perp} \approx \text{const}$).

The conclusion of the equation (2) is presented in an appendix.

Under the certain geophysical conditions and on a time interval approximately no more than three hours (when a geomagnetic activity index $Kp = \text{const}$) or on a greater time interval, when $Kp \approx \text{const}$, the equation (2) can be solved analytically.

Then the analytical solution of the equation (2) is

$$j_{\perp}(t) = j_{\perp 0} \exp \left(- \left(\frac{\pi \phi_2 L^3 \cos \phi}{7948800} + \frac{\gamma_{\perp} - \gamma_{\perp 0}}{\gamma_{\perp 0} (\gamma_{\perp 0} + 2) T_{wp}} \right) t \right). \quad (3)$$

Here, $j_{\perp 0}$ is the perpendicular differential flux of the charged particles at $t = 0$, ϕ_2 in [8], and ϕ is the azimuthal angle (local time $LT = 0$ h at midnight) or geomagnetic eastern longitude in the magnetic equator plane.

The conclusion of the equation (3) is presented in an appendix.

2. Experimental data and calculations

Further it will be used correlated observation of enhanced electromagnetic ion cyclotron (EMIC) waves and dynamic evolution of ring current proton flux collected by Cluster satellite near the location $L = 4.5$ during March 26 – 27, 2003, a nonstorm period ($D_{st} > -10$ nT) [2]. It is shown, that energetic (5 – 30 keV) proton fluxes are found to drop rapidly (e.g., a half hour) at lower pitch angles, corresponding to intensified EMIC wave activities.

For calculation we shall take following data [2]: $j_{\perp 0}(t = 0 \text{ s}) = 5.5894 \cdot 10^5 \text{ (cm}^2 \text{ s sr keV)}^{-1}$, $j_{\perp}(t = 1800 \text{ s}) = 2.1073 \cdot 10^5 \text{ (cm}^2 \text{ s sr keV)}^{-1}$, $Kp = 2.66$ or 3 -, $L = 4.17$, $\phi \equiv \text{MLT} = 22.58$ h, $\gamma_{\perp 0} = 0.5157$, $\gamma_{\perp} = 0.5801$, $E = 17.1$ keV. Then using (3) turn out $T_{wp} = 91.4988$ s and dependence j_{\perp} from t on the interval $t = (0 - 1800)$ s (Fig. 1).

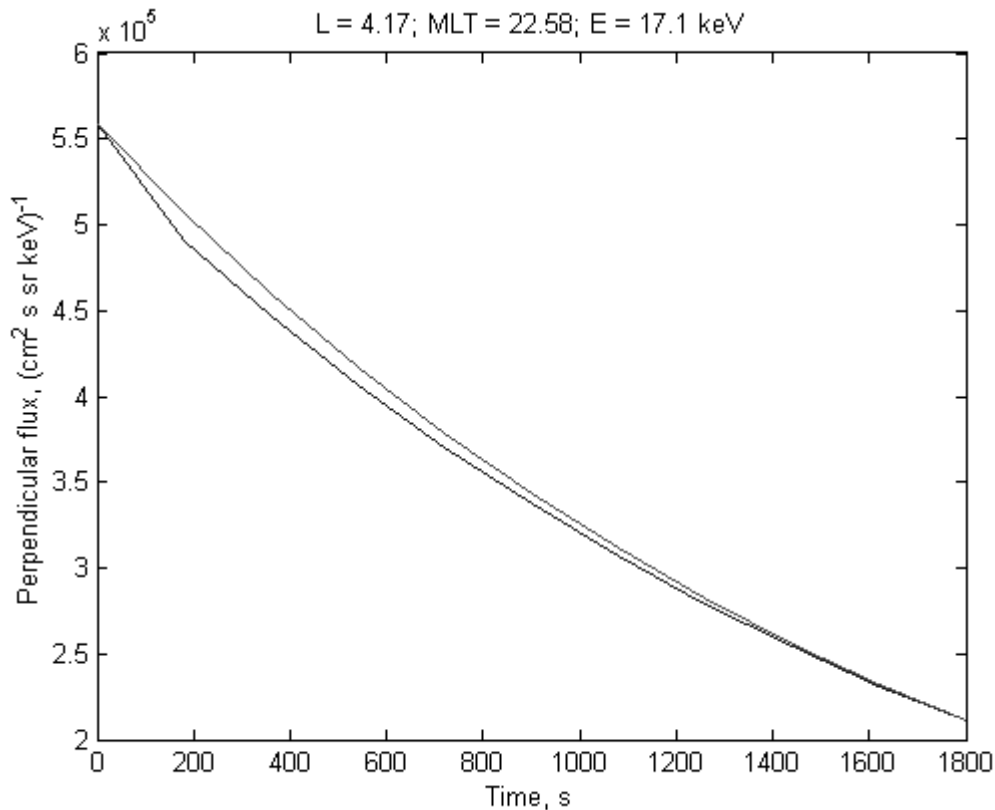


Рис. 1. Differential fluxes of protons for the 90° pitch angle on full model (1) – the bottom (blue) line and on the offered model (2) or (3) – the top (red) line.

Comparison of results on the offered model (2) or (3) and on full model (1) for the pitch angle

range from 0 up to 180 degrees is lead. For a perpendicular differential flux of the Earth's ring current protons very good consent with the maximal relative error approximately 3.23 % is received.

Conclusion

As mathematical model is offered the ordinary differential equation for the analytical description of a perpendicular differential flux of the charged particles in the Earth's magnetosphere which depends on time and several parameters.

The model allows to estimate also for different geophysical conditions a lifetime due to wave-particle interactions T_{wp} .

In the considered example for a perpendicular differential flux of the Earth's ring current protons very good consent with the maximal relative error approximately 3.23 % is received.

Appendix

A conclusion of the equations (2) and (3).

Considering, that $j = 2mEf$, $m = \text{const}$, $E = \text{const}$ and the coefficient of pitch angle diffusion is offered to be determined [3, 4] as follows

$$D_{\alpha\alpha} = D_{\perp} \sin^2 \alpha = \frac{1}{\gamma_{\perp 0} (\gamma_{\perp 0} + 2) T_{wp}} \sin^2 \alpha, \quad (\text{A1})$$

first from (1) we receive the equation

$$\frac{\partial j}{\partial t} = \frac{1}{\sin \alpha} \frac{\partial}{\partial \alpha} \left(\sin \alpha \cdot D_{\perp} \sin^2 \alpha \frac{\partial j}{\partial \alpha} - \frac{\sin^2 \alpha \cos \alpha}{2L} \frac{dL}{dt} j \right) - \frac{j}{T_{wp}} + S_{\perp} \sin^2 \alpha \cdot j. \quad (\text{A2})$$

Then we take derivatives and as a result it is received

$$\begin{aligned} \frac{\partial j}{\partial t} &= D_{\perp} \sin^2 \alpha \frac{\partial^2 j}{\partial \alpha^2} + \sin \alpha \cos \alpha \left(3D_{\perp} - \frac{1}{2L} \frac{dL}{dt} \right) \frac{\partial j}{\partial \alpha} + \\ &+ \left(- (2 \cos^2 \alpha - \sin^2 \alpha) \frac{1}{2L} \frac{dL}{dt} - \frac{1}{T_{wp}} + S_{\perp} \sin^2 \alpha \right) j. \end{aligned} \quad (\text{A3})$$

At $\alpha = 90^\circ$ and $\frac{\partial j}{\partial \alpha}|_{\perp} = 0$ the equation (A3) becomes simpler

$$\frac{dj}{dt}|_{\perp} \equiv \frac{dj_{\perp}}{dt} = D_{\perp} \frac{\partial^2 j}{\partial \alpha^2}|_{\perp} + \left(\frac{1}{2L} \frac{dL}{dt} - \frac{1}{T_{wp}} + S_{\perp} \right) j_{\perp}. \quad (\text{A4})$$

Proceeding from dependence $j = j_{\perp} \sin^{\gamma} \alpha$, $\gamma = \text{const} = \gamma_{\perp}$ it is possible to prove [7], that

$$\frac{\partial^2 j}{\partial \alpha^2}|_{\perp} = -\gamma_{\perp} j_{\perp}. \quad (\text{A5})$$

Then using [3, 4]

$$S_{\perp} = \frac{(\gamma_{\perp 0} + 3)}{(\gamma_{\perp 0} + 2) T_{wp}}$$

and (A1), (A4), (A5), the equation (2) \equiv (A6) turns out:

$$\begin{aligned} \frac{dj_{\perp}}{dt} + \left(-\frac{1}{2L} \frac{dL}{dt} + \frac{1}{T_{wp}} - S_{\perp} + D_{\perp} \gamma_{\perp} \right) j_{\perp} &= 0, \\ \frac{dj_{\perp}}{dt} + \left(-\frac{1}{2L} \frac{dL}{dt} + \frac{\gamma_{\perp} - \gamma_{\perp 0}}{\gamma_{\perp 0} (\gamma_{\perp 0} + 2) T_{wp}} \right) j_{\perp} &= 0. \end{aligned} \quad (\text{A6})$$

To receive the equation (3), we shall take the following approached equality [3, 4], using the bounce-averaged radial drift velocity of charged particles (measured in 1/s) in the Earth's magnetosphere

$$\frac{dL}{dt} \approx \left\langle \frac{dL}{dt} \right\rangle = -\Omega \frac{\phi_2}{\phi_0} L^4 \cos \phi = -\frac{2\pi \phi_2 L^4 \cos \phi}{24 \cdot 60 \cdot 60 \cdot 92}, \quad (\text{A7})$$

where ϕ is the azimuthal angle (local time $LT = 0$ h at midnight) or geomagnetic eastern longitude in the magnetic equator plane, Ω is the Earth's angular velocity, $\phi_0 = 92$ kV, and the dependence of ϕ_2 (measured in kV) on geomagnetic activity, i.e., on the Kp index, is determined from the formula [8]

$$\phi_2 = \frac{0.045}{(1 - 0.16Kp + 0.01Kp^2)^3}.$$

Believing all parameters in the equation (A6) constants (or approximately constants), we find the exact (or approached) analytical solution of the equation (A6) \equiv (2) in the form of (A8) \equiv (3):

$$j_{\perp}(t) = j_{\perp 0} \exp \left(- \left(\frac{\pi \phi_2 L^3 \cos \phi}{7948800} + \frac{\gamma_{\perp} - \gamma_{\perp 0}}{\gamma_{\perp 0} (\gamma_{\perp 0} + 2) T_{wp}} \right) t \right). \quad (\text{A8})$$

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