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EXACT SOLUTIONS OF THE CONFORMALLY FLAT UNIVERSE. I. THE EVOLUTION OF MODEL AS THE PROBLEM ABOUT A PARTICLE MOVEMENT IN A FORCE FIELD

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Abstract: The problem reduction of an evolution modelling of the open Universe for conformally flat space-time metric in Fock's form to an equivalent problem of a particle movement with an unit mass in a force field is demonstrated. The exact cosmological models filled with a substance and radiation in an approximation of the perfect fluid are found since the Friedman solution by means an introduction of set "mechanical" potentials.

In the article the possibility of deriving from the Einstein equations exact cosmological solutions for the open Universe by reduction to the equivalent problem of a mass particle motion in the force field is considered. The cosmological model is filled by substance in an approximation of the perfect fluid with nonzero pressure, generally speaking. The metric of 4D space-time is taken in the Fock form as the metric conformal to the Minkowski metric. This metric has the dependence on one variable. A square of the variable is product of advanced and retarded times.

The using of mechanical interpretation of the gravitation equations leads to a possibility of consideration of various mechanics force fields with the subsequent physical interpretation of the found exact cosmological solutions.

First of all a movement of a free particle with an unit mass (a mechanical force equals to zero) is considered, i.e. the particle moves on inertia. The fourth degree of discovered law of movement is a conformal factor of the cosmological metric which is conformally flat. This case corresponds to the exact cosmological solution without pressure, coinciding with known the Friedman solution for the open Universe.

After that the force field leading to uniformly decelerated motion of a particle is considered. The force potential is taken in the form of linear function. The tangent of a slope angle of the function curve coincides with particle acceleration. Such research leads to the exact cosmological solution asymptotically describing both an incoherent dust, and the ultrarelativistic substance which may be interpreted as an equilibrium radiation.

Further a square-law function without a linear term and a constant value is taken as a force potential. Such potential can be interpreted as potential of the free oscillator. The solution of corresponding equation of motion is written down in the form of a cosine function with some initial phase related to the ratio between parameters which define dust-like and ultrarelativistic substance. This conclusion becomes obvious after concidering asymptotic behaviour of pressure and energy density. Besides, the series expansion of a root of the fourth degree from a conformal factor asymptoticly coincides with the law of uniformly decelerated motion in previous case that indicates its particular character.

Keywords: open cosmological models, particle, Newton's equation, function of state.

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Introduction

Modern models of the Universe are based on the well-known cosmological solutions of Friedman [1,2] as on solutions of the Einstein equations. Without taking into account these models it is impossible to construct a realistic cosmological model. Up to today, the second solution [2] is the starting point for

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discussing cosmological models of the observable Universe for an isotropic space with negative curvature, describing an expanding universe filled with incoherent dust (there is no pressure).

At the same time, A. Friedman's solution [2] belongs to conformally flat solutions, i.e. for them the conformal curvature tensor of Weyl is equal to zero (see, for example, [3]). In this case we can write the metric of a four-dimensional space-time as a conformally flat 4D metric. In most cases, this solution of Friedman is written in the synchronous coordinates (see, for example, [4]). However, the Fock approach [5, 6] allows us to rewrite this metric in such form that is conformal to the Galilean metric (to the Minkowski metric). In addition, as shown under general approach in [7], that such a transition is equivalent to a transition from a synchronous frame of reference to a kinemetric one [8]– [14].

Further, a possibility to write the Einstein equations solution in the form of quadratures for a homogeneous isotropic Universe filled with matter with an arbitrary equation of state is obtained in the paper [15]. It is made with the help of the Fock approach, in which 4D metric of space-time is conformal to the Minkowski metric and the conformal factor is a function of one variable. Such writing down can be used to simulate the evolution of a model of the universe at various stages. But particular interest for the researcher there are exact solutions of the Einstein equations that generalize the already known ones.

The Fock approach applied in [16, 17] allowed us to find a generalization of Friedman's solution for the open universe in the case of both matter and equilibrium light-like radiation (similar to electromagnetic radiation) with non-zero pressure without introducing a specific equation of state. The continuation of this approach has been carried out in further works [18]– [21].

1. The equations of gravitational field

Now we will consider the possibility of obtaining exact solutions using the Fock approach. This procedure is different from earliar done in [16, 17].

Thus we assume that the metric may be written as:

$$ds^2 = \exp(2\sigma)\delta_{\mu\nu}dx^\mu dx^\nu, \tag{1}$$

where $exp(2\sigma)$ is a conformal factor; $\sigma = \sigma(S)$; $S^2 = \delta_{\mu\nu}x^\mu x^\nu = t^2 - r^2$; $\delta_{\mu\nu} = diag(1; -1; -1; -1)$ is the Minkowski metric tensor; $\mu, \nu = 0, 1, 2, 3$; the speed of light and Newton's gravitational constant are equal to unit, so the Einstein gravitational constant is equal here to $\varkappa = 8\pi$.

The right-hand side of the Einstein equations (without the cosmological constant)

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\varkappa T_{\mu\nu} \tag{2}$$

we will write in the approximation of the perfect fluid with an energy-momentum tensor (EMT)

$$T_{\mu\nu} = \varepsilon\, u_\mu u_\nu + p\, b_{\mu\nu}, \tag{3}$$

where ε is an energy density; p is a pressure; 4D-speed $u_\mu = exp(\sigma)b_\mu$ is proportional to the gradient of the variable S as a function of coordinates x^μ : $b_\mu = S_{,\mu}$; $u_\mu u^\mu = 1$ is 4D-speed normalization condition; $b_{\mu\nu} = u_\mu u_\nu - g_{\mu\nu}$ there is 3D-projector, which plays the role of 3D metric tensor and time-like congruence u^μ is normal to 3D-space: $b_{\mu\nu}u^\mu = 0$.

System of equations (2) is reduced to the system of two differential equations in full derivatives as a result (1+3)-splitting:

$$3\left(2\frac{\sigma'}{S} + (\sigma')^2\right) = \varkappa\varepsilon \cdot exp(2\sigma); \tag{4}$$

$$2\left(\sigma'' + \frac{2\sigma'}{S} + \frac{(\sigma')^2}{2}\right) = -\varkappa p \cdot exp(2\sigma), \tag{5}$$

where the prime denotes the derivative d/dS.

The splitting of the 4-equations of the gravitational field was carried out using the monad formalism [8]– [14] and projecting the system (2) onto the time-like world line and a space-like surface orthogonal to the time-like direction. The 4D-velocity vector is used as a monad. As a result, a system of two differential equations was obtained. One equation determines the energy density, and the other-the pressure.

Generally speaking, to solve this system, it is necessary to set the equation of state of matter, i.e. the relationship between energy density and pressure. This connection is obviously a function of the point of the space-time continuum (in this case it is a function of the variable S). The vast majority of cosmological models are constructed on the assumption that this relationship is a constant value, although it differs at different stages of the evolution of models. This assumption, in our opinion, is a barrier to the construction of truly evolutionary models, when matter (and, accordingly, the equation of state) changes because such models develop. We will try to demonstrate that, based on the form of the equations themselves, it is possible to obtain exact solutions with a variable equation of state that describe the Universe containing different stages of matter of its evolution.

First of all, we note that the system (4)–(5) can be reduced to the Riccati equation of the variable S by explicitly introducing a function of state as [15]

$$\beta(S) = \frac{p(S)}{\varepsilon(S)}. \tag{6}$$

On the assumption of a certain dependence of the pressure from the variable S [16, 17] the exact solution of this equation is found.

2. Reduction of the Universe simulation to the motion problem of a particle in a force field

Here we will take a slightly different path. By replacing $\sigma = 2 \cdot ln(y)$, the system (4)–(5) may be reduced to a simpler one

$$12 \cdot y' \left(y' + \frac{1}{S} \cdot y \right) = \varkappa\varepsilon \cdot y^6; \tag{7}$$

$$4 \cdot \left(y'' + \frac{2}{S} y' \right) = -\varkappa p \cdot y^5. \tag{8}$$

It is easy to see that in the equation (8) which contains pressure, one can exclude the first derivative by substituting:

$$y = z_1(1/S) \qquad or \qquad y = z_2(S)/S, \tag{9}$$

where z_1 and z_2 are some functions.

Here it is appropriate to draw an analogy with the external and internal tasks in potential theory, where the external solution (the Laplace equation solution) is often looked for in the first class functions, and the internal solution (the Poisson equation solution) in the second.

In this case, the equation (7) (determination of the energy density) for both cases of substitution will take the following form:

$$\frac{dz}{dx} \left(\frac{dz}{dx} - \frac{z}{x} \right) = \varkappa \frac{z^6}{12x^4} \cdot \varepsilon, \tag{10}$$

where $x = 1/S$ for the first replacement, and $x = S$ for the second one.

The equation (8) will be transformed to the general writing down in any case

$$\frac{d^2z}{dx^2} = F(x, z, p), \tag{11}$$

where

$$F(x, z, p) = -\varkappa \frac{z^5}{4x^4} \cdot p. \tag{12}$$

Considering the variable x as a new "time" variable, and the function z as a kind of generalized coordinate, we can interpret (11) as the Newton equation for the one-dimensional motion of a particle of unit mass under the action of the force F from equation (12).

We can integrate this equation knowing the function F, i.e. we come to the "law of motion" $y = y(x)$. In other words, in such way we can find the conformal factor $exp(2\sigma) = y^4$. This means that it is easy to find all physical magnitudes, which are necessary to us in the cosmological model. In this case, a specific "mechanical" movement of an unit mass particle will correspond to a specific Universe evolution. It is necessary to emphasize that the force function in mechanics can also depend on the velocity, for example, when we have an oscillator with dissipation (in particular, see obtaining a cosmological solution with a viscosity in [22, 23].

Thus, there is an opportunity *to replace the problem of modeling the evolution of the open Universe by the equivalent problem of the mechanical motion of an unit mass particle in a certain force field.*

The most common force fields are potential fields, therefore consideration of their here and in subsequent works will be devoted the further research. In particular, if $F = -dU/dz$, then the pressure is directly related to the choice of the function U dependence as

$$\varkappa p = 4\frac{x^4}{z^5} \cdot \frac{dU}{dz}. \tag{13}$$

We will stop here at the first replacement, i.e. we will introduce a new variable $x = 1/S$. Now we will mark the dimensionless "generalized coordinate" z as y. Thus everywhere below we will assume $z \equiv y$, because Friedman's solution for the open Universe just corresponds to such a dependence on S.

If the potential turns out to be a function of only "coordinate" z, then the equation (11) can be easily integrated using the corresponding "conservation law", in which "total energy" will be determined by the chosen initial conditions. This allows us to model different scenarios of the open Universe.

An inertial motion is the simplest example of motion for which $F = 0$ ($U \equiv U_0 = const$). Then, in such a "mechanistic" interpretation, the Friedman solution will correspond to an uniform motion or motion in a constant potential field:

$$p = 0; \quad \frac{dU}{dy} = 0; \quad y = 1 - Ax = 1 - \frac{A}{S}; \quad A > 0, \tag{14}$$

where A, on the one hand, is a constant "velocity" $\propto dy/dx$ (in the "mechanical" interpretation, where x is a "time variable", varying from $0 \Longleftrightarrow S = \infty$ to $1/A \Longleftrightarrow S = A$), and on the other hand, it is a constant associated with the density of dusty matter, i.e. matter that does not "create" pressure and fills the Friedman Universe.

The conformal factor of the metric (1) is here

$$exp(2\sigma) = y^4 = (1 - \frac{A}{S})^4 \tag{15}$$

and coincides with the one which is obtained in [5] for the open model of the Friedman Universe.

In [5, 6], the solution (14) is considered in sufficient details. Therefore, we will here not consider the analysis of Friedman's solution in such form of writing down. We will only remind of that the condition of the energy density positivity requires the positivity of the constant A, therefore we give here an exact expression for the energy density of the incoherent dust of the Friedman solution

$$\varkappa\varepsilon = \frac{12Ax^3}{(1 - Ax)^6}. \tag{16}$$

Obtaining the above result on the basis of the "mechanical" approach assumes that "non-uniform motion" in this approach should lead to a generalization of Friedman's solution.

The next simple "potential" is a linear function (with constants a and b),

$$U = ay + b, \tag{17}$$

i.e. uniformly decelerated motion is realized under the action of a constant force onto a unit mass particle:

$$F = -\frac{dU}{dy} = -a, \tag{18}$$

where a is a constant that coincides with the acceleration in the mechanical approach.

Then we immediately find dimensionless "path" by which this particle goes,

$$y = C_0 + Cx - \frac{a}{2}x^2, \tag{19}$$

where C and C_0 are the integration constants respectively interpreted as the velocity and the initial dimensionless distance when $x = 0$.

On the other hand, the expression (19) is the exact solution of the equations (7)–(8).

When $x \to 0$ ($S \to \infty$) the Galilean condition must be. In this case the Friedman solution as asymptotic solution must be satisfied for the open model of Universe. From these requirements we obtain the constants equal to $C_0 = 1$ and $C = -A$, in (19), i.e. we can rewrite the expression for y as

$$y = 1 - Ax - \frac{a}{2}x^2. \tag{20}$$

Such writing down of the function y with a positive parameter (a) sets up the condition $g_{00} < 1$ of a metric tensor component g_{00} for all $S < \infty$ or $x > 0$. In our case, this condition is true for the conformal factor, which be written as

$$exp(2\sigma) = \left(1 - Ax - \frac{a}{2}x^2\right)^4 = \left(1 - \frac{A}{S} - \frac{a}{2S^2}\right)^4. \tag{21}$$

It is clear, that the behavior of the model will be determined by the relationship between constants A and a. A complete study of this exact solution will be researched into later publications. Here we will indicate only some interesting, in our opinion, the points.

The pressure will be written as

$$\varkappa p(x) = \frac{4ax^4}{y^5}. \tag{22}$$

It should be said that when the constant a is negative, the pressure is also negative, i.e. the pressure sign is determined by the "acceleration" sign a.

The energy density from (10) is now written as

$$\varkappa\varepsilon(x) = 12x^3\frac{(A + ax)(1 + ax^2/2)}{y^6}. \tag{23}$$

The three-dimensional scalar curvature in this approach is calculated in accordance with the formula

$${}^3R = -\frac{3\sigma'_S}{S}exp(-2\sigma) = \frac{6x^3y'_x}{y^5} \tag{24}$$

and in our case it will be

$${}^3R = -\frac{6x^3(A + ax)}{(1 - Ax - ax^2/2)^5}. \tag{25}$$

The point of singularity in this model (the time-point when the conformal factor vanishes, and the three-dimensional scalar curvature equals to infinity) will correspond to the non-negativity of x

$$x_0 = 1/S_0 = \frac{-A + \sqrt{(A^2 + 2a)}}{a}. \tag{26}$$

In addition, the point of the singularity in the limit becomes equal to $x_0 = 1/A$ when the parameter a tends to zero ($a \to 0$).

The function of state (6) takes the form

$$\beta(x) = \frac{p(x)}{\varepsilon(x)} = \frac{1}{3}\frac{ax(1 - Ax - ax^2/2)}{(A + ax)(1 + ax^2/2)}. \tag{27}$$

It is easy to see from here that $\beta(x)$ vanishes on the ends of the segment: as in the point $x = 0$ so and into the singularity point x_0. And this is despite of the fact that into the point of the singularity, both the pressure and the energy density take on the infinite values.

The vanishing of $\beta(x)$ on the ends of the segment means that into some point in the segment the function of state reaches its maximum value, which depends on the ratio between the values of A and a. In a neighborhood of the point $x = 0$ for the state function the ratio is valid

$$\beta(x) \approx \frac{1}{3}\frac{a}{A}x. \tag{28}$$

In particular, if now we take $A = 0$ in the expression (27), i.e. assume that Universe is filled only with matter "creating" a non-zero pressure, we get the function

$$y = 1 - a^2x^2/2 \tag{29}$$

and the corresponding function of state

$$\beta = \frac{1}{3}\frac{\left(1 - a^2x^2/2\right)}{(1 + a^2x^2/2)}, \tag{30}$$

which will reach the maximum value $beta = 1/3$ when x tends to zero ($S \to \infty$).

This case will be is when the parameter A is small compared to a : $A << a$. It can be interpreted as the evolutionary "disappearance" of a dust substance, the existence of which is associated with the parameter A. At the same time, at the later stages of evolution, asymptotically we have matter with the equation of state of an ultrarelativistic gas. In other words, with the above-mentioned relationship between the parameters A and a, the maximum of the function β is shifted to the point $x = 0$.

In fact, we write down the corresponding pressure and energy density for the case $A = 0$:

$$\varkappa p = \frac{4ax^4}{(1 - ax^2/2)^5}; \tag{31}$$

$$\varkappa\varepsilon = 12ax^4\frac{(1 + ax^2/2)}{(1 - ax^2/2)^6}. \tag{32}$$

The asymptotic behavior of the expressions (31) and (32) nearby to $x = 0$ ($S \to \infty$) can be represented as

$$\varkappa p \approx 4ax^4 \tag{33}$$

and

$$\varkappa\varepsilon \approx 12ax^4. \tag{34}$$

This means that the not dusty matter filling the Universe, which "creates" a pressure different from zero and is associated with the parameter a, there is matter that asymptotically obeys the ultra-relativistic equation of state

$$\varepsilon_{rad} = 3p. \tag{35}$$

A similar consideration of the behavior of pressure (22) and energy density (23) nearby to $x = 0$ $(S \to \infty)$ leads to the following results:

$$\varkappa p \approx 4ax^4 \tag{36}$$

and

$$\varkappa\varepsilon \approx 12Ax^3(1 + 6Ax) + 12ax^4, \tag{37}$$

or

$$\varepsilon \approx \varepsilon_{dust} + \varepsilon_{rad}, \tag{38}$$

where ε_{dust} and ε_{rad} are, respectively, the energy density of the incoherent dust obtained asymptotically at $a = 0$ (coinciding with the asymptotic form of the relation (16)) and the energy density of the ultrarelativistic matter $(a \neq 0)$.

3. The open Universe model as an oscillator

Let's take the next step and consider the quadratic potential or oscillatory motion:

$$U = B^2y^2/2 + U_0, \tag{39}$$

where B is a constant that has the meaning of the spring stiffness coefficient in the mechanical interpretation.

From the equation (11), which takes the form of the equation "oscillations", we immediately get the exact solution given in [16], [17] and found there by more complex way as solving the Riccati equation:

$$y = \sqrt{(1 + A^2/B^2)}\,\cos(Bx + \alpha_0) = \frac{\cos(Bx + \alpha_0)}{\cos\,\alpha_0}, \tag{40}$$

where $\tan^2\alpha_0 = A^2/B^2 = 1 - 1/\cos^2\alpha_0,\ A = const,\ B = const.$

The conformal factor is written as

$$exp(2\sigma) = \left(1 + A^2/B^2\right)^2\cos^4\left(Bx + \alpha_0\right) = \left(1 + A^2/B^2\right)^2\cos^4\varphi(x) = \left(\frac{\cos\,\varphi(x)}{\cos\,\alpha_0}\right)^4, \tag{41}$$

where $\varphi \equiv B/S + \alpha_0 = Bx + \alpha_0$.

From (39) and (13), we immediately come to the expression for pressure:

$$\varkappa p = \frac{4B^2}{S^4}\frac{1}{\left(1 + A^2/B^2\right)^2\cos^4\varphi(S)} = 4B^2x^4\left(\frac{\cos\,\alpha_0}{\cos\,\varphi(x)}\right)^4, \tag{42}$$

and from (7) we have the expression for the energy density:

$$\varkappa\varepsilon = \frac{12B}{S^3}\frac{\tan\varphi(S)\left(1 + \dfrac{B}{S}\tan\varphi(S)\right)}{\left(1 + A^2/B^2\right)^2\cos^4\varphi(S)} = 12Bx^3\tan\varphi(x)\left(1 + Bx\tan\varphi(x)\right)\left(\frac{\cos\,\alpha_0}{\cos\,\varphi(x)}\right)^4. \tag{43}$$

From here it can be seen that the function of state takes the form:

$$\beta = \frac{1}{3}Bx\frac{\cot\varphi(x)}{(1 + Bx\tan\varphi(x))} \tag{44}$$

For this solution 3D scalar curvature is written as

$${}^3R = -6Bx^3\tan\varphi(x)\left(\frac{\cos\,\alpha_0}{\cos\,\varphi(x)}\right)^4 = -\frac{\varkappa\varepsilon}{2\left(1 + Bx\tan\varphi(x)\right)}. \tag{45}$$

To interpret the solution (41), it is necessary to consider the asymptotic behavior of the conformal factor, energy density, and pressure. We need to understand what the constants A and B are related to.

We limit our consideration to the interval $0 \le (Bx + \alpha_0) < \pi/2$. In other words, the singularity point (i.e. when the conformal factor vanishes, and the pressure and energy density tend to infinity) in our model it will correspond to

$$x_0 = 1/S_0 = \frac{(\pi/2 - \alpha_0)}{B}. \tag{46}$$

When x tends to zero ($S \to \infty$), the conformal factor naturally tends to unit.

In this case, for comparison with the observational data on density of matter and the radiation density in the modern era, we must associate the coordinate variable S with its own (physical) time as

$$\tau = \int\limits_{S_0}^{S} u_\mu dx^\mu = \int\limits_{S_0}^{S} y^2 dS = \left(1 + A^2/B^2\right) \int\limits_{S_0}^{S} \cos^2\left(\frac{B}{S} + \alpha_0\right) dS = -\frac{B}{\cos^2\alpha_0} \int\limits_{\pi/2}^{\varphi} \left(\frac{\cos\varphi}{\varphi - \alpha_0}\right)^2 d\varphi, \tag{47}$$

It is not difficult to see that asymptotically $\tau \approx S$. At the same time, an expansion of the pressure and energy density into a series of powers of x (or $(1/S)$) with accuracy up to $O(x^5)$ gives

$$\varkappa p \approx 4B^2x^4; \tag{48}$$

$$\varkappa\varepsilon \approx 12Ax^3(1 + 6Ax) + 12B^2x^4 = \varkappa\varepsilon_{dust} + \varkappa\varepsilon_{rad}, \tag{49}$$

where $\varkappa\varepsilon_{rad} = 3\varkappa p$ is the energy density of the ultrarelativistic state of matter, and $\varkappa\varepsilon_{dust}$ is the energy density of the incoherent dust, which equals to the asymptotic energy density of the Friedman solution in this approximation (see (16)).

From here it is clear that the constants (A) and (B) determine matter and radiation, respectively. The energy density in this case asymptotically splits into the direct sum of the dust energy density and the energy density of the equilibrium radiation (49). Moreover, the equation of state coincides (in this approximation) with the equation of state of an ultrarelativistic gas, which is not difficult to obtain by expanding into series the function of state (44).

But if the constant B is responsible for the presence of radiation in the model, then directing it to zero, we should get the Friedman solution in the limit. Indeed, in the limiting case when $B \to 0$, we obtain the Friedman solution (15) and thus make sure that the constant A from (41) is exactly Friedman's constant mentioned in the previous section.

$$\lim_{B\to 0}\left(\sqrt{(1 + A^2/B^2)}\cos\left(\frac{B}{S} + \alpha_0\right)\right) = 1 - A/S = 1 - Ax. \tag{50}$$

If we now expand the function $y(x)$ (40) in a series by degrees of x nearby the point $x = 0$ and limit ourselves to the accuracy of $O(x^3)$, we will actually get the expression

$$y(x) \approx 1 - Ax - \frac{B^2x^2}{2} \tag{51}$$

under the condition $a = B^2$.

Thus, the choice of the potential (39) lead to a generalization of the previously considered case.

Therefore, the solution under discussion is an exact solution of the Einstein equations, which describes the stages of the evolution of the Universe when there is matter and radiation (see (40), (41)). However, this conclusion follows from the asymptotic expressions. As for the earlier behavior of the model, it is necessary to investigate, for example, the ratio of pressure to energy density. This ratio is the equation of state in a small neighborhood of a given fixed value S.

Conclusion

In the paper is considered the possibility of obtaining exact cosmological solutions of the Einstein equations for the open Universe by reducing the problem to the equivalent problem of a motion of a particle with an unit mass in a force field. The cosmological model being studied is filled with matter in the approximation of the perfect fluid with a pressure not equal to zero, generally speaking. The metric of 4D space-time is chosen in the Fock form as a metric conformal to the Minkowski metric with a dependence on a single variable, the square of which is a product of the retarded and advanced times.

The use of a mechanical interpretation for one of the two equations of gravitation leads to the possibility of considering various mechanics force fields, in particular potential ones, with subsequent physical interpretation of the exact cosmological solutions obtained.

First of all, we consider the motion of a free particle of an unit mass (there is no any mechanical force), i.e., the particle moves according to the inertia law. The fourth degree of the found law of motion, taking into account the Galilean nature of space-time, is the conformal factor of the cosmological metric. This case corresponds to the exact cosmological solution without pressure, which coincides with the well-known Friedman solution for the open Universe.

Further we choose a force field leading to the uniformly decelerated motion of the particle, i.e. the corresponding force potential is selected in the form of a linear function and the tangent of the slope of the graph of the function coincides with the acceleration of the particle. After this study we have an exact cosmological solution that asymptotically describes both incoherent dust and ultrarelativistic matter, which could be interpreted as equilibrium radiation.

Next, a quadratic function without a linear term and a constant is chosen as the potential function. Such a potential can be interpreted as the potential of a free oscillator corresponding to a linear displacement force (the Hooke force). So the solution of the corresponding equation of motion is written down as a function of the cosine with some initial phase associated with the ratio of the parameters that determine the dusty and ultrarelativistic matter. This conclusion becomes apparent after an asymptotic consideration of the pressure and energy density. In addition, the series expansion of the fourth-degree root of the conformal factor coincides asymtotically with the law of uniformly decelerated motion in the previous case that indicates its particular character.

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