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ТОЧНЫЕ РЕШЕНИЯ В САМОГРАВИТИРУЮЩИХ SO(N)-ИНВАРИАНТНЫХ НЕЛИНЕЙНЫХ СИГМА-МОДЕЛЯХ.

Червон С. В.[a,b,c,1]

[a] Ульяновский государственный педагогический университет имени И. Н. Ульянова, Ульяновск, 432071, Россия.
[b] Московский государственный технический университет имени Н.Э. Баумана, Москва, 105005, Россия.
[c] Казанский Федеральный Университет, г. Казань, 420008, Россия.

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THE EXACT SOLUTIONS IN SELF-GRAVITATING SO(N)-INVARIANT NON-LINEAR SIGMA-MODELS

Chervon S. V.[a,b,c,1]

[a] Ulyanovsk State Pedagogical University, Ulyanovsk, 432071, Russia.
[b] Bauman Moscow State Technical University, Moscow, 105005, Russia.
[c] Kazan Federal University, Kazan, 420008, Russia.

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1. Introduction

Considering gravitational interaction of non-linear sigma-model (NSM) with a metric field $g_{ik}(x)$, satisfying the Einstein equations, the exact solutions of instanton and meron types of SO(N)-invariant non-linear σ-model on 4-dimensional Riemann spaces of the Euclidean signature were found in [1–3]. The general features of models and topological characteristics of solutions are studied.

Developing a such approach for Riemann spaces of the Lorentz signature (that helps to take into account the gravitational interaction between bosons, that have been described by (NSM) [4]) in [5,6] the classes of exact solutions SO(3) - SO(4)- invariant σ-model in spherical-symmetric and plane-symmetric spaces within the framework of cosmological models were found. Further generalization for the case of SO(N) internal symmetry is provided in this paper.

[1]E-mail: chervon.sergey@gmail.com

In Section 2 the scheme of generalization of plane-symmetric solutions (that obtained in [5, 6]) for the case of SO(N) internal invariance is proposed.

In Section 3 the method of solution‘s generation for SO(N)-invariant self-gravitating NSM from vacuum solutions of the Einstein equations is specified for the class of plane-symmetric spaces with the linear element

$$ds^2 = f(z,t)(-dz^2 + dt^2) + g_{ab}dx^a dx^b, \tag{1}$$

where f and g_{ab} depend on z and t only; $a, b = 1, 2$.

In Section 4 the vacuum metrics leading to the Friedmann cosmological solutions are found. It helps to find a new class of anisotropic cosmological models.

2. Plane-symmetric solutions in SO(N)-invatiant σ-model

The Lagrangian of self-gravitating σ-model is

$$\Lambda = \sqrt{-g}\left(R\varkappa^{-1} + \kappa^2 h_{AB}\varphi^A_{,i}\varphi^B_{,k}g^{ik}\right)/2, \tag{2}$$

where $h_{AB}(\varphi^C)$ is the metric of the value space of a multiplet of scalar (chiral) fields φ^C, κ^2 the constant of interactions with gravitational field, R the scalar curvature of coordinate space, $\varkappa$ the Einstein constant; $i, j, k = 1, 2, 3, 4$; $A, B, C = 1, ..., N-1$.

The energy momentum tensor (EMT) has components

$$T_{ik} = \kappa^2(h_{AB}\varphi^A_{,i}\varphi^B_{,k} - \frac{1}{2}g_{ik}h_{AB}\varphi^A_{,m}\varphi^B_{,n}g^{mn}). \tag{3}$$

Let us consider NSM with the value space

$$ds^2_\sigma = (d\varphi^1)^2 + \sin^2\varphi^1[(d\varphi^2)^2 + \sin^2\varphi^2[(d\varphi^3)^2 + ... + \sin^2\varphi^{N-2}(d\varphi^{N-1})^2]]$$

in the class of plane-symmetric spaces with the linear element (1) where

$$g_{12} = g_{21} = 0,\ \ g_{11} = g_{22} = A(z,t).$$

The Einstein's equations with the energy momentum tensor (3) is transformed to

$$\begin{aligned} A_{\xi\eta} = 0, \qquad \frac{1}{2A^2} + \frac{F_\xi}{A} = \kappa^2\varkappa < \varphi^1_\xi, \varphi^1_\xi >, \qquad F = \ln f, \\ \frac{1}{2A^2} + \frac{eF_\eta}{A} = \kappa^2\varkappa < \varphi^1_\eta, \varphi^1_\eta >, \qquad \frac{e}{2A^2} - F_{\xi\eta} = \kappa^2\varkappa < \varphi^1_\xi, \varphi^1_\eta > . \end{aligned} \tag{2.3}$$

In (2.3) and further we set up the designations: $z = \xi + \eta$, $t = \xi - \eta$, $\varphi_\xi = \partial_\xi\varphi$,

$$< \varphi^M_\xi,\ \varphi^M_\eta > \stackrel{def}{=} \varphi^M_\xi\varphi^M_\eta + \sin^2\varphi^M\Big[\varphi^{M+1}_\xi\varphi^{M+1}_\eta + \sin^2\varphi^{M+1}\big[\varphi^{M+2}_\xi\varphi^{M+2}_\eta + \ldots + \\ + \sin^2\varphi^{N-2}[\varphi^{N-1}_\xi\varphi^{N-1}_\eta]] \ldots\Big], \quad (M = 1, 2, \ldots, N-1).$$

Corresponding (2) field equations in considering case are:

$$\partial_\xi(A\varphi^1_\eta) + \partial_\eta(A\varphi^1_\xi) - A\sin 2\varphi^1 < \varphi^2_\xi,\ \varphi^2_\eta >= 0, \tag{2.4.1}$$

$$\partial_\xi(A\sin^2\varphi^1\cdot\varphi^2_\eta) + \partial_\eta(A\sin^2\varphi^1\cdot\varphi^2_\xi) - A\sin^2\varphi^1\sin 2\varphi^2 < \varphi^3_\xi,\ \varphi^3_\eta >= 0, \tag{2.4.2}$$

....

$$\partial_\xi(A\sin^2\varphi^1 ... \sin^2\varphi^{N-2}\times\varphi^{N-1}_\eta) + \partial_\eta(A\sin^2\varphi^1 ... \sin^2\varphi^{N-2}\times\varphi^{N-1}_\xi) = 0. \tag{2.4.N-1}$$

Taking into account (2.3) and using acceptable coordinate conversions, we have two options:

$$\text{A)}\ A = z,\ \ e = 1,$$

B) $A = t,\ e = -1.$

The case $A = A(\xi)$ (or $A = A(\eta)$) does not really interesting since it leads to a degeneration of the field equations (2.4.1)-(2.4.N-1).

1A. Let us suppose, that all fields are static: $\varphi^A = \varphi^A(z)$. Then, integrating (2.3), we have

$$f = |z|^{-\frac{1}{2}+\kappa^2\varkappa\alpha_0}, \quad (z < 0), \tag{2.5}$$

where the constant α_0^2 satisfies the relation

$$\alpha_0^2 = z^2 < \varphi_z^1,\ \varphi_z^1 >=< \varphi_u^1,\ \varphi_u^1 >, \quad (u = \ln|z|). \tag{2.6}$$

Taking into account (2.6) in (2.4) and recurrent connection

$$\begin{gathered} < \varphi_u^1,\ \varphi_u^1 >= \varphi_u^1\varphi_u^1 + \sin^2\varphi^1 < \varphi_u^2,\ \varphi_u^2 >, \\ < \varphi_u^2, \varphi_u^2 >= \varphi_u^2\varphi_u^2 + \sin^2\varphi^2 < \varphi_u^3, \varphi_u^3 > \\ \ldots\ldots\ldots\ldots\ldots \end{gathered} \tag{2.7}$$

the equation (2.4.1) is reduced to

$$(\varphi^1)_{uu} + \mathrm{ctg}\,\varphi^1\left[(\varphi_u^1)^2 - \alpha_0^2\right] = 0, \qquad \alpha_0^2 > (\varphi_u^2)^2, \tag{2.8}$$

and it has the solution

$$\varphi^1 = \arccos\left[\alpha_0^{-1}\sqrt{\alpha_0^2 - \alpha_1^2}\ \cos(\alpha_0 u)\right], \qquad \alpha_1^2 = \mathrm{const} < \alpha_0^2. \tag{2.9}$$

Taking into account (2.6)–(2.9), the equation (2.4.2) can be represented as

$$\sin^{-4}\varphi^1\left[(\varphi^2)_{\xi\xi} + \mathrm{ctg}\,\varphi^2\left((\varphi_\xi^2) - \alpha_1^2\right)\right] + \varphi_\xi^2\left[\xi_{uu} + \xi_u(\ln\sin^2\varphi^1)_u\right] = 0. \tag{2.10}$$

Requesting first and second terms are equal to zero in (2.10), we find

$$\varphi^2 = \arccos\left(\alpha_1^{-1}\sqrt{\alpha_1^2 - \alpha_2^2}\ \cos\alpha_1\xi\right), \qquad \xi = \alpha_1^{-1}\mathrm{arctg}(\alpha_1^{-1}\alpha_0\mathrm{tg}\alpha_0 u). \tag{2.11}$$

Taking into account (2.6)-(2.11), it is possible to reduce (2.4.3) and further equations to the form of (2.11). Directly integrating (2.4.N-1) we have

$$\begin{gathered} \varphi^1 = \arccos\left(\alpha_0^{-1}\sqrt{\alpha_0^2 - \alpha_1^2}\ \cos\xi_1\right), \qquad \xi_1 = \alpha_0 u, \\ \varphi^2 = \arccos\left(\alpha_1^{-1}\sqrt{\alpha_1^2 - \alpha_2^2}\ \cos\xi_2\right), \qquad \xi_2 = \mathrm{arctg}\left(\frac{\alpha_0}{\alpha_1}\mathrm{tg}\,\xi_1\right), \\ \ldots\ldots\ldots\ldots \\ \varphi^{N-2} = \arccos\left(\alpha_{N-3}^{-1}\sqrt{\alpha_{N-3}^2 - \alpha_{N-2}^2}\ \cos\xi_{N-2}\right), \qquad \xi_{N-2} = \mathrm{arctg}\left(\frac{\alpha_{N-4}}{\alpha_{N-3}}\ \mathrm{tg}\ \xi_{N-3}\right), \\ \varphi^{N-1} = \mathrm{arctg}\left(\frac{\alpha_{N-3}}{\alpha_{N-2}}\ \mathrm{tg}\,\xi_{N-2}\right), \qquad \alpha_0^2 > \alpha_1^2 > \ldots > \alpha_{N-2}^2. \end{gathered} \tag{2.12}$$

2A. Suppose, that the chiral fields φ^A depend on time only: $\varphi^A = \varphi^A(t)$. From equations (2.3) we find:

$$f = |z|^{-\frac{1}{2}}\exp\left\{\kappa^2\varkappa\alpha_0^2\frac{z^2}{2}\right\}, \qquad A = z. \tag{2.13}$$

The solution of (2.4) is similarly to the case 1A and it has the view as (2.12) with the substitution $u \to t$.

1B. Let $\varphi^A = \varphi^A(t)$. Then we have

$$f = |t|^{\varkappa\kappa^2\alpha_0^2 - \frac{1}{2}}, \qquad A = t < 0. \tag{2.14}$$

The solution for (2.14) looks like (2.12), where $u = \ln|t|$.

2B. Let $\varphi^A = \varphi^A(z)$.

$$f = |t|^{-\frac{1}{2}} \exp\left\{\varkappa\kappa^2\alpha_0^2 \frac{t^2}{2}\right\}, \qquad A = z. \tag{2.15}$$

The solutions of fields equations (2.4) looks like (2.12) where $u = z$.

Let us note, that obtained gravitational fields (2.5), (2.13)–(2.15) for SO(N)-invariant self-gravitating NSM have the same form like in papers [5,6]. Beside that, knowing the gravitational field of the resulted classes of solutions it is impossible to determine the number of chiral fields generating these solutions, because the metric (and the components of EMT (2.2)) contains only one constant — α_0^2.

3. Solutions' generating in self-gravitating SO(N)-invariant σ-model from vacuum solutions of Einstein's equations

Following by the work [7], let us represent the metric coefficient f in the metric (1) as multiplication $f = f_{\mathrm{v}} \cdot \Phi$, where f_{v} is the solution of Einstein's vacuum equations $R_{ik} = 0$; $\Phi(z,t)$-correction factor for the matter (in our case – for multiplet of scalar fields φ^A). Then the equations of self-gravitating SO(N)-invariant NSM is disintegrated into four groups.

The first group of equations:

$$(\alpha \hat{g_{,\xi}} \hat{g}^{-1})_\eta + (\alpha \hat{g_{,\eta}} \hat{g}^{-1})_\xi = 0,$$

where $\alpha^2 = \det(g_{ab})$, $\hat{g} = (g_{ab})$, $\hat{g}\hat{g}^{-1} = E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

The second group:

$$(\ln f_{\mathrm{v}})_\xi = \frac{(\ln\alpha)_{\xi\xi}}{(\ln\alpha)_\xi} + \frac{Sp\hat{A}^2}{4\alpha\alpha_\xi},$$

$$(\ln f_{\mathrm{v}})_\eta = \frac{(\ln\alpha)_{\eta\eta}}{(\ln\alpha)_\eta} + \frac{Sp\hat{B}^2}{4\alpha\alpha_\eta},$$

where $\hat{A} = -\alpha \hat{g_{,\xi}} \hat{g}^{-1}$, $\hat{B} = \alpha \hat{g_{,\eta}} \hat{g}^{-1}$ — 2×2 matrices.

The third group — the field equations (2.4) taking into account replacement A for α.

The fourth group determines the correction for scalar fields factor Φ:

$$(\ln\Phi)_\xi = \varkappa\kappa^2 \frac{< \varphi^1_\xi, \varphi^1_\xi >}{(\ln\alpha)_\xi}, \quad (\ln\Phi)_\eta = \varkappa\kappa^2 \frac{< \varphi^1_\eta, \varphi^1_\eta >}{(\ln\alpha)_\eta}. \tag{3.1}$$

The integrability conditions of (3.1) are fulfilled by virtue of the field equations (2.4). Indeed, by making a linear combination

$$(2.4.1) \times \varphi^1_\xi + (2.4.2) \times \varphi^2_\xi + \ldots + (2.4.N-1) \times \varphi^{N-1}_\xi,$$

we have got the integrability conditions (3.1).

Thus, for getting the exact solutions of NSM in spaces (1.1), it is enough to solve the field equations (2.4) for vacuum metric and to find from (3.1) the correction factor Φ.

The solutions of Section 2 can be considered as generated from vacuum ones, besides for considered cases Φ and f_{v} are:

1A. $\Phi = |z|^{\varkappa\kappa^2\alpha_0^2}, \quad f_v = f_0^2 |z|^{-\frac{1}{2}}, \quad A = z, \quad f_0^2 = \text{const}.$

2A. $\Phi = \exp\left\{\varkappa\kappa^2\alpha_0^2 \cdot z^2/2\right\}, \quad f_v = f_0^2 |z|^{-\frac{1}{2}}, \quad A = z.$

1B. $\Phi = |t|^{\varkappa\kappa^2\alpha_0^2}, \quad f_v = f_0^2 |t|^{-\frac{1}{2}}, \quad A = t.$

2B. $\Phi = \exp\left\{\varkappa\kappa^2\alpha_0^2 \cdot t^2/2\right\}, \quad f_v = f_0^2 |z|^{-\frac{1}{2}}, \quad f_v = f_0^2 |t|^{-\frac{1}{2}}, \quad A = t.$

Let us note, that the correction factor Φ for the scalar fields φ^A, according to (3.1), is determined up to a constant factor: $\Phi \to \text{const} \cdot \Phi$.

4. Cosmological solutions of SO(N)-invariant σ-model

As the initial vacuum solutions we choose the metric classes, which lead to Friedmann cosmological models with scalar field [7].

The linear element

$$ds^2 = \operatorname{sh} 2t \left[f_0^2 \left(\frac{\operatorname{sh} 4\xi \cdot \operatorname{sh} 4\eta}{\operatorname{sh}^2 2t} \right)^{\tilde{\kappa}^2} (-dt^2 + dz^2) + \operatorname{sh}^2 z \, dx^2 + \operatorname{ch}^2 z \, dy^2 \right], \tag{4.1}$$

where $0 \le t < \infty$, $0 \le z < \infty$, $0 \le x \le 2\pi$, $-\infty < y < \infty$, $f_0^2, \tilde{\kappa}^2$ – const., is the solution of the Einstein‘s vacuum equations $R_{ik} = 0$ and it leads to the open cosmological model.

The initial linear element for closed cosmological model is:

$$ds^2 = \sin 2t \left[f_0^2 \left(\frac{\sin 4\xi \cdot \sin 4\eta}{\sin^2 2t} \right)^{\tilde{\kappa}^2} (-dt^2 + dz^2) + \sin^2 z \, dx^2 + \cos^2 z \, dy^2 \right], \tag{4.2}$$

where $0 \le z \le \frac{\pi}{2}$, $0 \le x \le 2\pi$, $-\pi \le y \le \pi$.

The initial metric for the flat world has the view

$$ds^2 = t \left[f_0^2 \left(\xi \eta^{t-2} \right)^{\tilde{\kappa}^2} (-dt^2 + dz^2) + z^2 dx^2 + dy^2 \right], \tag{4.3}$$

where $0 \le z < \infty$, $0 \le x \le 2\pi$, $-\infty < y < +\infty$.

There is no metric classes (4.1)–(4.3) in [7], but there is the procedure of getting them from Friedmann standard solutions (with scalar field included).

Let us generate the solutions SO(N)-invariant NSM, using the described in Section 3 method, from the vacuum solution (4.1).

1^0. Let us suppose that the chiral fields depend on time only: $\varphi^A = \varphi^A(u)$, where $u = \frac{1}{2} \ln |\text{th t}|$. Then the field equations (2.4) have the solution (2.12), where $u = \frac{1}{2} \ln |\operatorname{tg} t|$.

Integrating (3.1) we find the coefficient

$$\Phi = (\operatorname{sh}^2 2t \cdot \operatorname{sh}^{-1} 4\xi \cdot \operatorname{sh}^{-1} 4\eta)^{\tilde{\kappa}^2}, \quad \tilde{\kappa}^2 = \frac{4}{3} \varkappa\kappa^2\alpha_0^2. \tag{4.4}$$

Taking into account (4.4) we get the open isotropic universe in metrics‘ class (1.1) (in (4.1) we suppose $\tilde{\kappa}^2 = 0, \ f_0^2 = 1$).

2^0. Let us suppose that $\varphi^A = \varphi^A(u)$ where $u = \frac{1}{2} \ln |\operatorname{tg} z|$. Then the solution of (2.4) has the view (2.12) where $u = \frac{1}{2} \ln |\operatorname{tg} z|$, $< \varphi_u^1, \ \varphi_u^1 >= \alpha_0^2$. Integrating (3.1), we determine the correction factor for matter Φ:

$$\Phi = (\operatorname{sh}^2 2z \cdot \operatorname{sh}^{-1} 4\xi \cdot \operatorname{sh}^{-1} 4\eta)^{\tilde{\kappa}^2}, \quad \tilde{\kappa}^2 = 4\varkappa\kappa^2\alpha_0^2 / 3.$$

As the result we get the anisotropic cosmological metric

$$ds^2 = \operatorname{sh} 2t \left[f_0^2 (\operatorname{sh} 2t \ \operatorname{sh}^{-1} 2t)^{\tilde{\kappa}^2} (-dt^2 + dz^2) + \operatorname{sh}^2 z \, dx^2 + \operatorname{ch}^2 z \, dy^2 \right]. \tag{4.5}$$

Thus, in the case of dependence of scalar fields φ^A on space coordinate z, SO(N)-invariant NSM generates the anisotropic cosmological model (4.5).

Based on vacuum solutions (4.2),(4.3) we find that in case of time-like chiral fields φ^A, with analogical assumptions, SO(N)-invariant NSM generates the closed and flat universe (in (4.2), (4.3) set $\tilde{\kappa}^2 = 0,\ f_0^2 = 1$). In the case of dependence of φ^A on space coordinate z we get the anisotropic cosmological models:

$$ds^2 = \sin 2t \left[f_0^2 \left(\frac{\sin 2z}{\sin 2t} \right)^{\tilde{\kappa}^2} (-dt^2 + dz^2) + \sin^2 z\, dx^2 + \cos^2 z\, dy^2 \right], \qquad (4.6)$$

$$d^2 = t \left[f_0^2 (zt^{-1})^{\tilde{\kappa}^2} (-dt^2 + dz^2) + z^2 dx^2 + dy^2 \right], \quad \tilde{\kappa}^2 = \frac{4}{3} \varkappa \kappa^2 \alpha_0^2, \qquad (4.7)$$

which are creating by the multiplet of scalar fields (2.12), where $u = \frac{1}{2} \ln |\mathrm{tg}\, 2z|$ for solution (4.6) and $u = \frac{1}{2} \ln |z|$ for solution (4.7).

5. Conclusion

Thus, the scheme of plane-symmetric solutions generalization of self-gravitating σ-model for the case of SO(N) internal invariance (Sec.2), using the solution generating for σ-model from vacuum metric method (Sec.3), let us not only to interpret the solutions obtained earlier as the solutions generated from vacuum solutions, but to find new classes of solutions describing the anisotropic cosmological models (Sec.4) as well. Also it lets us to show that SO(N)-invariant NSM can be the source for Friedmann cosmological models of open, closed and flat universe.

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Авторы

Червон Сергей Викторович, д.ф.-м.н., профессор, профессор кафедры физики и технических дисциплин, УлГПУ им. И.Н. Ульянова, площадь Ленина, 4/5, г. Ульяновск, 432071, Россия. Профессор кафедры физики МГТУ им. Н.Э. Баумана, 2- ая Баумановская ул., д. 5, стр. 1, г. Москва, 105005, Россия. Ведущий научный сотрудник, Институт физики, КФУ, ул. Кремлевская, д. 18, г. Казань, 420008, Россия.
E-mail: chervon.sergey@gmail.com

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Authors

Chervon Sergey Victorovich, Doctor of Physical and Mathematical Sciences, Professor, Department of Physics and Technical Disciplines, Ulyanovsk State Pedagogical University, Lenin's square, Build. 4/5, Ulyanovsk, 432071, Russia. Professor of Physics Department of Bauman Moscow State Technical University, 2-nd Baumanskaya street, 5, Moscow, 105005, Russia. Leading Researcher of Kazan Federal University, Kremlevskaya street 18, Kazan, 420008, Russia.
E-mail: chervon.sergey@gmail.com

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