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О МОДИФИКАЦИИ УРАВНЕНИЙ ЭЙНШТЕЙНА НА МАЛЫХ РАССТОЯНИЯХ (НА АНГЛИЙСКОМ) *

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ON THE MODIFICATION OF EINSTEIN'S EQUATION AT SMALL DISTANCES (IN ENGLISH)

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1. The equations of the modified theory of gravity

Suppose that the geometric properties of spacetime at small distances can be described within the framework of Riemannian geometry with the metric tensor $g_{ij}(x)$, and the distributions of matter and other physical fields, except for the gravitational one, can be described using the energy-momentum tensor $T_{ij}(x)$. Since in some coordinate system, where x^4 is a timelike coordinate, T_{44} is the energy density, it is possible to transform it into quantities having only the dimension of length in two ways: a) using the Einstein constant $\varkappa = 8\pi\gamma/c^4$, b) using the constant $8\pi f/\hbar c$, where $f \sim 1$. In the first case, $\varkappa T_{44}$ has the dimension of length to the power of -2, in the second case, to the power of -4.

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In the first case, the geometry of spacetime is determined by Einstein's equations

$$G_{ij} = \varkappa T_{ij}. \tag{1}$$

In the second case, the equations must be of the fourth order, and it follows from them that the covariant divergence of the energy-momentum tensor $T_{ij}(x)$ is equal to zero.

Apparently, the simplest equation satisfying the specified condition has the form:

$$G_{ij,\,kp} g^{kp} = \frac{8\pi f}{\hbar c} T_{ij}(x). \tag{2}$$

In the future, we will denote the operator $g^{kp}\nabla_k \nabla_p \equiv \Box$.

It is also possible to write an equation that includes equations (1) and (2)

$$\frac{\hbar c}{8\pi f}\,\Box\, G_{ij} + \frac{c^4}{8\pi\gamma} G_{ij} = T_{ij}. \tag{3}$$

It is quite possible that the appearance of terms with higher-order derivatives is not accidental, since in quantum field theory there exists a regularization method using higher covariant derivatives (for example, in gauge field theory [1]). It is also possible that the equations have an infinite number of terms containing the operator $\Box$. For example,

$$\ldots + \left(\frac{\gamma\hbar}{c^3 f}\right)^n \Box^n G_{ij} + \ldots + \frac{\gamma\hbar}{c^3 f}\Box G_{ij} + G_{ij} = \varkappa T_{ij} \tag{4}$$

or in the general case

$$\psi(\Box) G_{ij} = \varkappa T_{ij}, \tag{5}$$

where $\psi(\Box)$ is some analytic function of the operator $\Box$.

A theory containing derivatives of infinite order will essentially be non-local.

In the future, we will focus on the analysis of equations (2) and (3). Dimensional analysis of equation (3) shows that the first term begins to play a role at distances close to the Planck scale. At larger scales, it becomes insignificant, and the equation becomes equivalent to the usual Einstein equations.

At the same time, equation (2) contains only Planck's constant, and in this regard an interesting question arises: can equation (2) describe the geometry of spacetime at the level of elementary particles and their interactions?

2. Cosmological Solutions of Modified Equations in the Class of Spatialy Flat Friedmann Metrics

The line element of the spatially flat Friedmann metric has the form:

$$ds^2 = -a^2(T)\left(dx^2 + dy^2 + dz^2\right) + dT^2. \tag{6}$$

The field equations (3) for this metric reduce to one differential equation for the scale factor a:

$$\frac{\hbar c}{8\pi f}\left[2\frac{da}{dT}\cdot\frac{d^3a}{dT^3} + 2\left(\frac{d^2a}{dT^2}\right)^2 - \frac{4}{a^2}\left(\frac{da}{dT}\right)^4\right] + \frac{c^4}{8\pi\gamma}\left(\frac{da}{dT}\right)^2 = \frac{1}{3}a^2\varepsilon(a). \tag{7}$$

Let us assume that the scale factor reaches an extreme value at some point. Without loss of generality, we can assume that the extreme point is reached at the moment $T = 0$. At this point, $da/dT = 0$. From equation (7), it follows that at this point the second derivative has the value

$$\frac{d^2a}{dT^2}(0) = \pm\sqrt{\frac{4\pi f}{3\hbar c} a_0^2 \varepsilon(a_0)}\ . \tag{8}$$

From (8) it follows that there can exist solutions reaching the minimum or maximum value of the scale factor. The behavior of the solution near the extremum is described by the formula

$$a = a_0 \pm a_0 \sqrt{\frac{\pi f}{3\hbar c}\varepsilon_0(a_0)}\; T^2. \tag{9}$$

Formula (9) shows that the nonsingular solution (9) is associated with the presence of the first term in the left-hand side of equation (3), and it actually determines the nature of the solution's approach to the extreme point.

In the general case, for real equations of state:

$$\varepsilon(a) = \varepsilon_0 \left(\frac{a_0}{a}\right)^n, \tag{10}$$

where $3 \le n \le 4$. If $n = 3$, the matter represents dust; if $n = 4$, the matter has an ultrarelativistic equation of state. At $n = 10/3$, equation (7) admits an exact solution, which, however, is singular at $T = 0$:

$$a = a_0 \left(\frac{5}{3}\right)^{3/5} \left(\frac{8\pi\gamma\varepsilon_0}{3c^4}\right)^{3/10} T^{3/5}. \tag{11}$$

For system (2), we also obtain a single equation for determining the scale factor:

$$2\frac{da}{dT}\frac{d^3a}{dT^3} + 2\left(\frac{d^2a}{dT^2}\right)^2 - \frac{4}{a^2}\left(\frac{da}{dT}\right)^4 = \frac{8\pi f}{3\hbar c}a^2\varepsilon(a), \tag{12}$$

which, when $\varepsilon \neq 0$, also admits an approximate nonsingular solution (9).

Equation (12) has, in contrast to Einstein's equations (1), the following nontrivial vacuum solutions:

$$a(T) = a_0 e^{-bT}. \tag{13}$$

This solution is the de Sitter solution:

$$a(T) = a_0 T^{3/5}. \tag{14}$$

Solution (14) is singular and coincides in form with solution (11). The question of the existence of exact regular solutions remains open.

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