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WEAK GRAVITATIONAL WAVES AND PETROV CLASSIFICATION

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***Abstract*:** It is considered the problem of a superposition of the Weyl matrices with different canonical bases as sum these matrices with a point of view of Petrov's algebraic classification of gravitational fields. Weyl matrices are close connected with the algebraic classification. Such superposition of the Weyl matrices has physical interpretation in superposition of weak gravitational fields and may be used for getting resulting gravitational field. An example of an investigation there is sum of two Weyl matrices for two gravitational plane waves of type N by Petrov classification. In linear approximation we get a new resulting solution of the Einstien equations with traceless energy-momentum tensor which is nilpotent matrix of index three. The energy-momentum tensor of the electromagnetic high frequency radiation is the nilpotent matrix of index two. The optical expansion scalars; the optical scalars describing rotation and shear of new congruences in resulting gravitational field vanish. Thus the congruence with tangent eigenvector of energy-momentum tensor in the first approximation behaves as a laminary flow of perfect fluid similarly as free electromagnetic radiation.

***Keywords*:** gravitational waves, algebraic classification of Petrov, Weyl matrices, a superposition (composition) principle of space types, linearised Einstein equations.

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Introduction

The algebraic classification of spaces proposed by Petrov [1] permits to study gravitational fields without taking into account how specifically there fields were "derived"from some given gravitational fields. On the other hand, algebraic Petrov classification is related closely to phase transitions between types of gravitational fields [2–4]. Also a composition of Petrov's algebraic types on the Weyl matrices level can be constructed (see [5], [6]).

We consider here the connection of resulting weak gravitational field with initial weak fields (neglecting their mutual interactions), the canonical frames of 3×3 complex traceless symmetrical Weyl matrices being linked by infinitesimal rotations. Different canonical frames of such Weyl matrices make up the Petrov algebraic classification of space-times (algebraic classification of gravitational fields). Obviously the superposition of these fields satisfies linearised Einstein equations.

1. Principle of superposition and Weyl matrices

Each gravitational field type corresponds to the concrete type of Weyl matrices for the Petrov algebraic classification. Thus into account of this remark the principle of superposition for two weak gravitational fields can be expressed by a sum of Weyl matrices, which must be written in the same orthonormal basis with help of matrices of rotation T in complex 3D space,

$$W_3 = W_1^c + T_2 W_2^c \tilde{T}_2, \tag{1.1}$$

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where W^c is the Weyl matrix in canonical form, and T is an orthonormal complex matrix, $T^{-1} = \tilde{T}$ is a transposed matrix.

It is known orthonormal complex matrix always is represented as (see [7])

$$T = R\, exp\,(i\, K),$$

where i is an imaginary unit, $i^2 = -1$, R is an orthonormal real matrix and K is an antisymmetric real matrix, $\tilde{K} = -K$.

Matrix T is connected with the Lorents transformation in 4D tangent space in the form of (see [8])

$$T_{ik} = 2\Omega_k^{\alpha\beta} L_{i\alpha} L_{0\beta}; \qquad \alpha, \beta = 0, 1, 2, 3;$$

$$\Omega_k^{\alpha\,\beta} = \delta^{\alpha}_{[k}\delta^{\beta}_{0]} - \frac{i}{2}\varepsilon_{kmn}\delta^{\alpha}_m\delta^{\beta}_n; \qquad k, m, n = 1, 2, 3. \tag{1.2}$$

where δ^{β}_{α} is Kronecker symbol; $\varepsilon_{k\,m\,n}$ is 3D Levi-Civita symbol, square brackets denote antisymmetrisation (symmetrisation will be expressed as $(a\,b)$).

Weyl matrix can be now written in the form

$$W_{i\,k} = \overset{(+)}{W}_{k\,0\,j\,0} = \frac{1}{2}\Omega_k^{\alpha\,\beta}\Omega_j^{\gamma\,\delta} W_{\alpha\beta\gamma\delta}, \tag{1.3}$$

where

$$\overset{(+)}{W}_{\alpha\beta\gamma\delta} = W_{\alpha\beta\gamma\delta} - i\, W^{*}_{\alpha\beta\gamma\delta}$$

is the self dual Weyl tensor with

$$\overset{(+)}{W}{}^{*}_{\alpha\beta\gamma\delta} = i\overset{(+)}{W}_{\alpha\beta\gamma\delta},$$

and

$$W^{*}_{\alpha\beta\gamma\delta} = \frac{1}{2}\,\varepsilon_{\alpha\beta\mu\nu}\, W_{\mu\nu\gamma\delta}$$

is the dual Weyl tensor; the duality operation marks as $*$, $\varepsilon_{\alpha\beta\mu\nu}$ is 4D Levi-Civita symbol.

The Weyl conformal curvature tensor can be written as

$$W_{\alpha\beta\gamma\delta} = R_{\alpha\beta\gamma\delta} + R_{\gamma[\alpha} g_{\beta]\delta} - R_{\delta[\alpha} g_{\beta]\gamma} + \frac{1}{3} R\, g_{\gamma[\alpha} g_{\beta]\delta}. \tag{1.4}$$

where $R_{\alpha\beta\gamma\delta}$ is the Riemann tensor of cuvature; $R_{\alpha\beta}$ is the Ricci tensor; R is the scalar curvature, and $g_{\alpha\beta}$ is a metrical tensor connected closely with a space-time metric which is a quadric form

$$ds^2 = g_{\alpha\beta} dx^{\alpha} dx^{\beta}. \tag{1.5}$$

The Weyl tensor (1.4) is a base of a construction of 3×3 complex traceless symmetrical Weyl matrices with a help of mapping (6).

The set of orthonormal matrices T as functions of complex parameters $(\varphi_j) = (\varphi_1, \varphi_2, \varphi_3)$ form a continuous group, the infinitesimal transformations of which being skew symmetrical matrices

$$A_k = \left(\frac{\partial T}{\partial \varphi_k}\right)_{|\varphi_i = 0}; \qquad \tilde{A}_k = -A_k. \tag{1.6}$$

Let the matrix W be connected with certain matrix W_0 by similarity transformation,

$$W = T W_0 \tilde{T}, \tag{1.7}$$

where $T = T(\varphi_1, \varphi_2, \varphi_3)$, and W_0 is independent of parameters φ_j.

Differentiation of expression (13) with respect to φ_k at the point $\varphi_i = 0$ with making use of (12) gives

$$(W_{,k})_0 \equiv \left(\frac{\partial W}{\partial \varphi_k}\right)_0 = [A_k, W_0], \tag{1.8}$$

where $[A, B] = AB - BA$ is a commutator of matrices.

It is easy to find now

$$W_{,j,k} = [A_k, [A_j, W_0]] + [B_{kj}, W_0]. \tag{1.9}$$

Here

$$B_{kj} = \left(\frac{\partial^2 T}{\partial\varphi_j \partial\varphi_k}\right)_0 - A_k A_j.$$

Higher order derivatives can be found by a similar procedure.

It can be shown that the rank of the matrix being a function of parameter, may be changed by differentiation. Therefore ranks of the matrices (1.8), (1.9) will perhaps not be equal to rank of the initial matrix (13). If each Weyl matrix W is supposed to correspond to a gravitational field of certain metric, then the change of the rank of W (leading to the corresponding change of the Petrov algebraic type) means a transition to a new gravitaional field of another metric. Hence one can consider the expression (1.8), for instance, as a way of "derivation" of a new gravitational field from a primary one. In this case the gravitational field was not supposed to be weak.

We consider now the group of rotation in 3D the Euclidean complex space. Corresponding matrices are

$$T = exp(\varphi_j X_j). \tag{1.10}$$

These matrices are the set of orthonormal matrices as functions of complex parameters (φ_j)=$(\varphi_1, \varphi_2, \varphi_3)$ in 3D complex space (summation over repeated indices from 1 to 3 is supposed). Furthermore matrices T form a continuous group of matrices, where the matrices

$$X_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}; \quad X_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}; \quad X_3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{1.11}$$

are generators of infinitesimal rotations around corresponding axes and skew symmetrical matrices

$$X_k = \left(\frac{\partial T}{\partial\varphi_k}\right)_{|\varphi_j=0}; \qquad \tilde{X}_k = -X_k. \tag{1.12}$$

Further taking φ_j as small parameters we express W_3 as a series

$$W_3 = W_1^c + W_2^c + [A_i, W_2^c]\,\varphi_i + \frac{1}{2}[A_i\,[A_k, W_2^c]\,]\,\varphi_i\varphi_k + ... \tag{1.13}$$

with the use of formulas (1.8), (1.9), (1.10); $B_{ik} = 0$ and $T_2 = T_2(\varphi_i)$ from (3).

This decomposition represents the resulting gravitational field as a sum of fields of given Petrov algebraic types with extra terms due to the choice of non-canonical basis of one of the two summands. The question can now be posed, the gravitational fields of which types are to be added to the sum $W_1^c + W_2^c$ in order to get, up to the desired degree of approximation, the gravitational field correspponding to W_3.

On the other hand, a given field can be considered as a superposition of two initial fields plus small corrections.

Now we express (3) as

$$W_3^c = \tilde{T}_3 W_1^c T_3 + \tilde{T}_3 T_2 W_2^c \tilde{T}_2 T_3, \tag{1.14}$$

and for the series in small parameters' powers we have

$$W_3^c = W_1^c + W_2^c - [B_i, (W_1^c + W_2^c)]\,\eta_i + [A_i, W_2 c]\,\varphi_i - \frac{1}{2}[B_i, [B_k, (W_1^c + W_2^c)]]\,\eta_i\eta_k + [B_i, [A_k, W_2^c]]\,\varphi_k\eta_i + ..., \tag{1.15}$$

where $B_i = ((T_3)_{,i})_0$ and $T_3 = T_3(\eta_i)$.

It is worth mentioning that if $W_1^c \equiv 0$, the decomposition (1.15) describes the non-canonical basis choice, but by an additional rotation it is possible regain the canonical frame, e.g. by a transition from one frame of reference to another.

2. Weak gravitational fields and Weyl matrices

Taking the Weyl matrix of the algebraic type N (a wave type) now in its canonical basis,

$$W_1^c = W_2^c = W_N^c = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & i \\ 0 & i & -1 \end{pmatrix}; \tag{2.1}$$

one may symbolically rewrite the decomposition (1.13) when $\varphi_1 = 0$, $\varphi_2 = 0$, $\varphi_3 \equiv \varphi$ as

$$W = 2N + III\,\varphi + II\,\varphi^2 + Ia\,\varphi^3 + ... \tag{2.2}$$

(after the commutators were found and their Petrov algebraic types were determined). In symbolic decomposition (2.2) W_N^c (see (2.1)) is marked here as N.

The matrix of algebraic type III in canonical form (also a wave type)

$$W_{III}^c = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -i \\ 0 & -i & 0 \end{pmatrix} \tag{2.3}$$

is marked by symbol of III.

The symbol of II corresponds to algebraic type of II with canonical matrix

$$W_{II}^c = \begin{pmatrix} -2a & 0 & 0 \\ 0 & a+1 & i \\ 0 & i & a-1 \end{pmatrix}, \tag{2.4}$$

where a is a parameter.

Further algebraic type Ia with canonical matrix

$$W_{Ia}^c = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{2.5}$$

is marked as symbol of Ia.

If parameters in decomposition (1.13) $\varphi_2 = \varphi_3 = 0$, then the matrix W belongs to type N, since this rotation does not destroy the canonical basis.

The symbolic decomposition (2.2) is similar to one of the peeling-off theorem by Sachs (see e.g. [9] p.131, [10] and [11]), but the parameter φ plays here the role of rotation angle connecting two canonical frame of two matrices (both frame are taken at the same point of the manifold). From this decomposition it is also clear that a superposition of two gravitational fields of type N with non coinciding frame does not give the resulting field of type N.

We consider now the possibility to find (in the linear approximation) a new gravitational field, departing from a weak plane gravitational wave in vacuum (algebraic type N) with the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{2.6}$$

where the quantities $h_{\mu\nu} = h_{\mu\nu}(x^0 - x^1)$ and their derivatives are infinitesimals of the first order; $h_{\mu\nu} = diag(1, -1, -1, -1)$; $h_{22} = -h_{33}$; $h_{23} = h_{32}$. The Weyl tensor in this case coincides with the curvature tensor. It is known that this metric satisfies the Hilbert-Lorentz transversality conditions, and null Killing vector exists, $\xi_\mu = \delta_\mu^0 - \delta_\mu^1$ giving the direction of propagation of the wave in 4D space.

It is easy to find the connection of the new matrix with the initial one by determinig the Weyl matrix for the initial matric, making use of the expression (1.8) (with $\varphi = \varphi_3$, $A = X_3 = X$), and indentifying the commutator $[X, W]$ and the new $\hat{W}$. For the non-vanishing components we have

$$\hat{W}_{12} = \hat{W}_{21} = \hat{W}_{22}; \quad \hat{W}_{13} = \hat{W}_{31} = \hat{W}_{23}. \tag{2.7}$$

The matrix $\hat{W}$ belongs to Petrov type III, and it can brought to the canonical form by the similarity transformation with

$$T = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.8}$$

with $\det T = 1$.

Though this procedure does not influence the variables on which components of the Weyl tensor depends, the vacuum Einstein equations do not hold for the new metric. Straightforward calcutations show that if the metric in linear approximation depends only on the retarded time, and the linearized Einstein equations in vacuum are satisfied, then the space can be of type N only. Therefore we use in the following calculations the Weyl conformal curvature tensor (1.4) in linear approximation.

Linearized expressions for curvature tonsor, Ricci tensor and scalar curvature can be written in terms of the derivatives of $\hat{h}_{\mu\nu}$ as follows ($\hat{h}_{\mu\nu}$ depends on retarded time $u = x^0 - x^1$):

$$\hat{R}_{\beta\gamma} = \hat{h}^{\alpha}{}_{(\gamma,\beta),\alpha} - \frac{1}{2}\hat{h}_{\,,\gamma,\beta}\,; \tag{2.9}$$

$$\hat{R} = \hat{h}^{\alpha\beta}{}_{,\alpha\beta}\,; \tag{2.10}$$

$\hat{h} = \hat{h}^{\alpha}{}_{\alpha}$; $\hat{h}_{\mu\nu}$ describes a new gravitational field. Components of the matrix $\hat{W}$ are expressed through components of the Weyl tensor as

$$\hat{W}_{12} = \hat{W}_{1020} + i\,\hat{W}_{2023}\,; \quad \hat{W}_{13} = \hat{W}_{1030} + i\,\hat{W}_{3023}\,. \tag{2.11}$$

In the linear approximation

$$\hat{W}_{1020} = \hat{R}_{1020} + \frac{1}{2}\hat{R}_{12} = \frac{1}{4}(\ddot{\hat{h}}_{\,02} + \ddot{\hat{h}}_{\,12})\,; \tag{2.12}$$

$$\hat{W}_{2023} = \hat{R}_{2023} - \frac{1}{2}\hat{R}_{30} = -\frac{1}{4}(\ddot{\hat{h}}_{\,30} + \ddot{\hat{h}}_{\,31})\,; \tag{2.13}$$

$$\hat{W}_{1030} = \hat{R}_{1030} - \frac{1}{2}\hat{R}_{13} = \frac{1}{4}(\ddot{\hat{h}}_{\,03} + \ddot{\hat{h}}_{\,13})\,; \tag{2.14}$$

$$\hat{W}_{3023} = \hat{R}_{3023} - \frac{1}{2}\hat{R}_{02} = \frac{1}{4}(\ddot{\hat{h}}_{\,02} + \ddot{\hat{h}}_{\,12}), \tag{2.15}$$

where a dot means retarded time derivative.

By virtue of (1.10) and (1.13) the new and old functions $h_{\mu\nu}$ obey the relations

$$2h_{22} = \hat{h}_{02} + \hat{h}_{12}\,; \quad 2h_{23} = \hat{h}_{03} + \hat{h}_{13}\,; \tag{2.16}$$

(here integration constants are put equal to zero and only the components $\hat{h}_{02}$, $\hat{h}_{12}$, $\hat{h}_{03}$, $\hat{h}_{13}$, are suppoted to be non zero).

The relations (2.16) clearly do not fix unambiguously the functions $\hat{h}_{\mu\nu}$, so that several variants of their choice exist, for instance

$$\hat{h}_{02} = \hat{h}_{12} = \hat{h}_{22}\,; \quad \hat{h}_{03} = \hat{h}_{13} = \hat{h}_{23}\,; \tag{2.17}$$

$$\hat{h}_{02} = \hat{h}_{03} = 0\,; \quad \hat{h}_{12} = 2\hat{h}_{22}\,; \quad \hat{h}_{13} = 2\hat{h}_{23}\,; \tag{2.18}$$

and so on.

The metric $\hat{g}_{\mu\nu}$ satisfies Einstein equations with a traceless energy-momentum tensor [12]

$$T_{\mu\nu} = m_{\mu}\,l_{\nu} + l_{\mu}\,m_{\nu} \tag{2.19}$$

with

$$m_\mu = b\,\delta^2_\mu + a\,\delta^3_\mu\,; \quad l_\mu = \delta^0_\mu - \delta^1_\mu\,; \quad m_\mu m^\mu < 0\,; \quad l_\mu l^\mu = 0\,; \quad m_\mu l^\mu = 0\,; \quad b = -\frac{1}{8\pi}\ddot{\hat{h}}_{22}\,; \quad a = -\frac{1}{8\pi}\ddot{\hat{h}}_{23}$$

and $T^\mu{}_\mu = 0$, $T^\mu{}_{\nu,\mu} = 0$ (in this approximation).

This energy-momentum tensor may be written in matrix block form as

$$(T_{\mu\nu}) = \begin{pmatrix} 0 & C \\ \tilde{C} & 0 \end{pmatrix}, \tag{2.20}$$

where matrix C is

$$C = \begin{pmatrix} b & a \\ -b & -a \end{pmatrix} \tag{2.21}$$

with $\det C = 0$.

We investigate now the tensor $T_{\mu\nu}$ independently of its origin. Consider the eigenvalue problem

$$T^\mu_\nu Y^\nu = \lambda\, Y^\mu, \tag{2.22}$$

where Y^ν are the eigenvectors, and $\lambda_{(\nu)}$ are eigenvalues.

All eigenvalues $\lambda_{(\nu)}$ are equal to zero, and eigenvectors (in this case there is no timelike one) are

$$Y^\mu = l^\mu + n^\mu\,, \tag{2.23}$$

here $n^\mu n_\mu < 0$; $n^\mu = a\,\delta^\mu_2 - b\,\delta^\mu_3$; $n^\mu l_\mu = n^\mu m_\mu = 0$.

Hence $T_{\mu\nu}$ is energy-momentum tensor of some null field (on the properties of energy-momentum tensor of the null electromagnetic field see e.g. [9], p.65).

Furthermore energy-momentum tensor as matrix (see (2.20)) is a nilpotent matrix of index 3, i.e. $(T_{\mu\nu})^3 = 0$. The energy-momentum tensor of the electromagnetic high frequency radiation in the form of $T_{\mu\nu} \propto l_\mu l_\nu$ is the nilpotent matrix of index 2. And else, in accordance to classification of Plebański [13] tensor $T_{\mu\nu}$ from (2.19) belongs to degenerate third type and such energy-momentum tensor can not describe macro distribution of matter.

It is worth mentioning that the optical expansion scalars of the congruences with tangent vectors l^μ and n^μ vanish,

$$\varepsilon = \frac{1}{2}\,l^\mu{}_{;\mu} = \frac{1}{2}\,n^\mu{}_{;\mu} = 0, \tag{2.24}$$

so that $Y^\mu{}_{;\mu} = 0$ (in the considered approximation).

The optical scalars describing rotation and shear of these congruences vanish too. Thus the congruence with tangent vector Y^μ in the first approximation behaves as a laminary flow of perfect fluid (free electromagnetic radiation behaves likewise).

Conclusion

In the paper the superposition of Weyl matrices with different canonical bases as sum these matrices with a point of view of Petrov's algebraic classification of gravitational fields is considered. Weyl matrices are close connected with the algebraic classification of spaces. At first, the superposition of the Weyl matrices has physical interpretation in superposition of weak gravitational fields and may be used for getting resulting gravitational field. At second, such superposition of the Weyl matrices gives new Weyl matrix of concrete algebraic type corresponding to a new gravitational field. All this it is demonstrated at example of sum two Weyl matrices with wave type N. As result we have in linear approximation the metric of the new gravitational field with traceless energy-momentum tensor which is nilpotent matrix of index three. Here we must say that energy-momentum tensor of the electromagnetic high frequency radiation is the nilpotent matrix of index two. Moreover the optical expansion scalars, the optical scalars describing rotation and shear of new congruences in resulting gravitational field vanish. And congruence of with tangent eigenvector of energy-momentum tensor in the first approximation behaves as a laminary flow of perfect fluid likewise of free electromagnetic radiation.

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