УДК 53.1

КИРАЛЬНАЯ КОСМОЛОГИЧЕСКАЯ МОДЕЛЬ $F(R, \Box R)$ ГРАВИТАЦИИ

Червон С. В.[a,b,1], Фомин И. В.[a,b,2], Чаадаева Т. И.[a,3]

[a] Ульяновский государственный педагогический университет имени И.Н. Ульянова, г. Ульяновск,432071, Россия

[b] Московский государственный технический университет имени Н.Э. Баумана, г. Москва, 105005, Россия

Изучаем модифицированную гравитацию $f(R, \Box R)$, которая может быть сведена к киральной космологической модели специального типа с тремя скалярными полями χ, ϕ, φ. Метод сведения к киральной космологической модели использовался для модели $f(R, (\nabla R)^2)$, но изначально был применен Naruko для более общей модели $f(R)$ гравитации. В данной работе представлены уравнения для киральных полей и уравнения Эйнштейна-Фридмана в метрике Фридмана-Робертсона-Уокера. Отметим, решения для данной выбранной модели строятся на специальном выборе одного из киральных полей: $\chi = -\sqrt{\frac{3}{2}}\ln 2$. Рассматриваются решения в случае сведения модели к однополевой модели: два киральных поля модели зависят линейно от третьего. Полученные решения представляют собой интерес, так как при анализе необходимо учитывать и квантовые поправки.

Ключевые слова: киральная космологическая модель, скалярно-тензорная и $f(R)$ теория гравитации, вселенная Фридмана, космологические параметры, спектральные параметры.

CHIRAL COSMOLOGICAL MODEL OF $F(R, \Box R)$ GRAVITY

Chervon S. V.[a,b,1], Fomin I. V.[a,b,2], Chaadaeva T. I.[a,3]

[a] Ulyanovsk State Pedagogical University, Ulyanovsk, 432071, Russia

[b] Bauman Moscow State Technical University, Moscow, 105005, Russia

We study the modified gravity $f(R, \Box R)$, which can be reduced to a chiral cosmological model of a particular type with scalar field observers χ, ϕ, φ. The method of studying the chiral cosmological model was used for the $f(R, (\nabla R)^2)$ models, but was first used by Naruko for the more general $f(R)$ Gracing model. In this paper, equations for chiral fields and the Einstein-Friedmann equations in the Friedmann-Robertson-Walker metric are presented. Note that the solutions for this chosen model are based on a special choice of one of the chiral fields: $\chi = -\sqrt{\frac{3}{2}}\ln 2$. Solutions are considered in the case of reducing the model to a one-field model: two chiral fields of the model depend linearly on the third one. The obtained solutions are of interest, since quantum corrections must also be taken into account in the analysis.

Keywords: chiral cosmological model, scalar-tensor and $f(R)$ gravity theory, Friedman universe, cosmological parameters, spectral parameters.

PACS: 04.20.Kd, 04.50.Kd
DOI: 10.17238/issn2226-8812.2023.1.117–120

Introduction

The need of modification of Einstein gravity closely connected with discovery of the acceleration in the expansion of the Universe. After this discovery it became clear that GR could not explain this

[1]E-mail: chervon.sergey@gmail.com
[2]E-mail: ingvor@inbox.ru
[3]E-mail: majorova.tatyana@mail.ru

phenomena by natural way without introduction of additional fields (dark energy). Therefore there were studied modifications of gravity theory such as the Einstein-Gauss-Bonnet theory, scalar-tensor theory of gravity, $f(R)$ gravity and $f(R)$ gravity with high derivatives.

1. Chiral Cosmological Model of $f(R, \Box R)$ gravity

The method of transformation of the model $f = f(R, \Box R)$ to Einstein gravity with scalar fields, according to the method described in [1] for the model $f(R, (\nabla R)^2, \Box R)$, is represented in the paper [2] in detail. The final form of the action after such transformation in Einstein frame is [2]:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\chi_{,\mu}\chi_{,\nu} - \frac{1}{2}e^{-\sqrt{\frac{2}{3}}\chi}g^{\mu\nu}\varphi_{,\mu}\phi_{,\nu} + \frac{1}{4}e^{-2\sqrt{\frac{2}{3}}\chi}\left(f(\phi,\varphi) - \varphi B(\phi,\varphi)\right) - \right.$$
$$\left. - \frac{1}{4}e^{-\sqrt{\frac{2}{3}}\chi}\phi\right], \tag{1.1}$$

where $f(\phi,\varphi)$ and $B(\phi,\varphi)$ are the functions defining concrete form of the model.

The action integral (1.1) can be considered as three-component chiral cosmological model with fields $\{\chi(t), \phi(t), \varphi(t)\}$, the target space metric with non-zero components $h_{11} = 1$, $h_{23} = \frac{1}{2}e^{-\sqrt{\frac{2}{3}}\chi}$ and the potential of interaction $W(\chi,\phi,\varphi) = \frac{1}{4}e^{-\sqrt{\frac{2}{3}}\chi}\phi - \frac{1}{4}e^{-2\sqrt{\frac{2}{3}}\chi}\left(f(\phi,\varphi) - \varphi B(\phi,\varphi)\right)$.

From the action (1.1) by standard variation we obtain the chiral fields equations

$$\ddot{\chi} + 3H\dot{\chi} + \frac{1}{\sqrt{6}}e^{-\sqrt{\frac{2}{3}}\chi}\dot{\phi}\dot{\varphi} - \frac{1}{2\sqrt{6}}e^{-\sqrt{\frac{2}{3}}\chi}\left(\phi - 2e^{-\sqrt{\frac{2}{3}}\chi}\left[f - \varphi B\right]\right) = 0, \tag{1.2}$$

$$\ddot{\varphi} + 3H\dot{\varphi} - \sqrt{\frac{2}{3}}\dot{\chi}\dot{\varphi} + \frac{1}{2}\left(1 - e^{-\sqrt{\frac{2}{3}}\chi}\left[f_{,\phi} - \varphi B_{,\phi}\right]\right) = 0, \tag{1.3}$$

$$\ddot{\phi} + 3H\dot{\phi} - \sqrt{\frac{2}{3}}\dot{\chi}\dot{\phi} - \frac{1}{2}e^{-\sqrt{\frac{2}{3}}\chi}\left[f_{,\varphi} - B - \varphi B_{,\varphi}\right] = 0, \tag{1.4}$$

and Einstein – Friedman equations

$$3H^2 = \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}e^{-\sqrt{\frac{2}{3}}\chi}\dot{\phi}\dot{\varphi} + \frac{1}{4}e^{-\sqrt{\frac{2}{3}}\chi}\phi - \frac{1}{4}e^{-2\sqrt{\frac{2}{3}}\chi}\left(f(\phi,\varphi) - \varphi B(\phi,\varphi)\right), \tag{1.5}$$

$$\dot{H} = -\frac{1}{2}\dot{\chi}^2 - \frac{1}{2}e^{-\sqrt{\frac{2}{3}}\chi}\dot{\phi}\dot{\varphi}. \tag{1.6}$$

2. Solutions for the selected model

To simplify the search of model's solutions we study the case when the scalar field χ is equal to the special constant $\chi = -\sqrt{\frac{3}{2}}\ln 2$. This value of χ corresponds to identical conformal transformation [3], [4]. For this case, the system of equations of the model can be easily obtained from (1.2)-(1.6).

We analyzed the following cases:

1) When we set potential $W(\phi,\varphi)$ equal to zero: $W(\phi,\varphi) = \frac{1}{2}\phi - \left(f(\phi,\varphi) - \varphi B(\phi,\varphi)\right) = 0$, the case which is valid for rather large class of $f(R, \Box R)$ models of gravity. the solution is

$$H = \frac{1}{3(t - t_*)}, \tag{2.1}$$

$$\varphi = C_\varphi \ln(t - t_*) + \varphi_*, \tag{2.2}$$

$$\phi = C_\phi \ln(t - t_*) + \phi_*. \tag{2.3}$$

2) When we set the potential equal to constant: $W_* = 3H^2 + \dot{H}$. Then is possible to find a connection between chiral fields. The fields equation will be the same for $W = W_*$. Therefore we find the following solutions

$$H = \sqrt{\frac{W_*}{3}}\tanh(\sqrt{3W_*}(t-t_*)), \quad W_* > 3H^2, \quad \dot{H} > 0, \tag{2.4}$$

$$\phi = \frac{C_\phi}{\sqrt{3W_*}}\sinh\sqrt{3W_*}(t-t_*) + \phi_*, \tag{2.5}$$

$$\varphi = \frac{C_\varphi}{\sqrt{3W_*}}\sinh\sqrt{3W_*}(t-t_*) + \varphi_*. \tag{2.6}$$

The second solution for $W_* < 3H^2$:

$$H = \sqrt{\frac{W_*}{3}}\coth(\sqrt{3W_*}(t-t_*)), \quad W_* < 3H^2, \quad \dot{H} < 0, \tag{2.7}$$

$$\phi = C_\phi \ln\left|\tanh\left(\frac{\sqrt{3W_*}}{2}(t-t_*)\right)\right| + \phi_*, \tag{2.8}$$

$$\varphi = C_\varphi \ln\left|\tanh\left(\frac{\sqrt{3W_*}}{2}(t-t_*)\right)\right| + \varphi_*. \tag{2.9}$$

If $W_* < 0$ we have $\dot{H} < 0$. Therefore we have the solution:

$$H = -\sqrt{-\frac{W_*}{3}}\tan\left(\sqrt{-3W_*}(t-t_*)\right), \tag{2.10}$$

$$\phi = \frac{C_\phi}{2\sqrt{-3W_*}}\ln\left|\frac{1+\sin\left(\sqrt{-3W_*}(t-t_*)\right)}{1-\sin\left(\sqrt{-3W_*}(t-t_*)\right)}\right| + \phi_*, \tag{2.11}$$

$$\varphi = \frac{C_\varphi}{2\sqrt{-3W_*}}\ln\left|\frac{1+\sin\left(\sqrt{-3W_*}(t-t_*)\right)}{1-\sin\left(\sqrt{-3W_*}(t-t_*)\right)}\right| + \varphi_*. \tag{2.12}$$

Once again one can see the linear dependence of chiral fields ϕ and φ.

Following by procedure introduced in the article [5] to apply the standard method of cosmological parameters calculation (power spectrum, spectral indexes, tensor-to-scalar ratio) we reduce three-field model to single field inflationary model assuming linear dependence between fields:

$$\phi(t) = k_\phi\chi(t), \quad \varphi(t) = k_\varphi\chi, \quad k_\phi = const, \quad k_\varphi = const. \tag{2.13}$$

Analysis of obtained parameters shows the necessity to include quantum corrections (say as Gauss-Bonnet term) into consideration.

Conclusion

The paper considers a chiral cosmological model of the form $f(R,\Box R)$, as well as some solutions found for the model. The material presented in the article is of interest for further analysis of the results obtained and the search for approaches to match the model with observational data.

References

1. Naruko A., Yoshida D., Mukohyama S. Gravitational scalar-tensor theory. *Class. Quant. Grav.*, 2016, vol. 33, no. 9, pp. 09LT01.

2. Chervon S.V., Fomin I.V., Chaadaev A.A. Spherically Symmetric Solutions of a Chiral Self-Gravitating Model in $f(R,\Box R)$ Gravity. *Grav. Cosmol.*, 2022, 28, no. 3, pp. 296–304.

3. Saridakis E.N., Tsoukalas M. Cosmology in new gravitational scalar-tensor theories. *Physical Review D.*, 2016, vol. 93, no. 12, pp. 124032.

4. Chervon S.V., Fomin I.V., Mayorova T.I. Chiral Cosmological Model of $f(R)$ Gravity with a Kinetic Curvature Scalar. *Grav. Cosmol.*, 2019, 25, no. 3, pp. 205–212.

5. Chervon S.V., Fomin I.V., Mayorova T.I., Khapaeva A.V. Cosmological parameters of f(R) gravity with kinetic scalar curvature. *J. Phys. Conf. Ser.*, 2020, 1557, 012016.

Авторы

Червон Сергей Викторович, д.ф.-м.н., профессор, кафедра физики, Ульяновский государственный педагогический университет имени И.Н. Ульянова, пл. Ленина, д. 4/5, Ульяновск, 432071, Россия; Факультет физики, Московский государственный технический университет имени Н.Э. Баумана (национальный исследовательский университет), ул. 2-я Бауманская, д. 5, Москва, 105005, Россия.
E-mail: chervon.sergey@gmail.com

Фомин Игорь Владимирович, д.ф.-м.н., профессор, факультет физики, Московский государственный технический университет имени Н.Э. Баумана (национальный исследовательский университет), ул. 2-я Бауманская, д. 5, Москва, 105005, Россия; кафедра физики, Ульяновский государственный педагогический университет имени И.Н. Ульянова, пл. Ленина, д. 4/5, Ульяновск, 432071, Россия.
E-mail: ingvor@inbox.ru

Чаадаева Татьяна Игорьевна, лаборатория гравитации, космологии, астрофизики, Ульяновский государственный педагогический университет имени И.Н. Ульянова, пл. Ленина, д. 4/5, Ульяновск, 432071, Россия.
E-mail: majorova.tatyana@mail.ru

Просьба ссылаться на эту статью следующим образом:
Червон С. В., Фомин И. В., Чаадаева Т. И. Киральная космологическая модель $f(R, \Box R)$ гравитации. *Пространство, время и фундаментальные взаимодействия.* 2023. № 1. С. 117–120.

Authors

Chervon Sergey Viktorovich, Ph.D., Professor, Department of Physics, Ulyanovsk State Pedagogical University, Lenin's square, 4/5, Ulyanovsk, 432071, Russia; Physics Department, Bauman Moscow State Technical University, 2-nd Baumanskaya street 5, Moscow, 105005, Russia.
E-mail: chervon.sergey@gmail.com

Fomin Igor Vladimirovich, Ph.D., Professor, Physics Department, Bauman Moscow State Technical University, 2-nd Baumanskaya street 5, Moscow, 105005, Russia; Department of Physics, Ulyanovsk State Pedagogical University, Lenin's square, 4/5, Ulyanovsk, 432071, Russia.
E-mail: ingvor@inbox.ru

Chaadaeva Tatyana Igorevna, Laboratory of gravitation, cosmology, astrophysics, Ulyanovsk State Pedagogical University, Lenin's square, 4/5, Ulyanovsk, 432071, Russia.
E-mail: majorova.tatyana@mail.ru

Please cite this article in English as:
Chervon S. V., Fomin I. V., Chaadaeva T. I. Chiral cosmological model of $f(R, \Box R)$ gravity. *Space, Time and Fundamental Interactions*, 2023, no. 1, pp. 117–120.