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ВЛИЯНИЕ КИНЕТИЧЕСКОГО СКАЛЯРА КРУЧЕНИЯ НА КОСМОЛОГИЧЕСКИЕ РЕШЕНИЯ $F(T)$ ГРАВИТАЦИИ*

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В данной работе рассматривается степенная инфляция в рамках модифицированной телепараллельной гравитации с высшими производными по скаляру кручения вида $F(T,(\nabla T)^2)$. Параметр Хаббла выражается как $H = \frac{m}{t}$, где $m > 1$. Анализируютя уравнения космологической динамики для частных значений $\omega(T)$ при выборе функции F: $F = T + \omega(T)\nabla_\mu T\nabla^\mu T$. Показано, что энергия и давление темного сектора, возникающие за счет высших производных, ослабевают на больших временах. На раннем этапе эволюции Вселенной они поддерживают степенную инфляцию, причем зависимость потенциала $V(\phi)$ от скалярного поля ϕ обоснована с физической точки зрения.

Ключевые слова: Модифицированная телепараллельная гравитация, степенная инфляция, темная энергия.

THE INFLUENCE OF THE KINETIC SCALAR OF TORSION ON THE COSMOLOGICAL SOLUTIONS OF F(T) GRAVITY

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This work considers power-law inflation within the modified teleparallel gravity with higher-derivative torsion terms of the kind $F(T,(\nabla T)^2)$. The Hubble parameter is expressed as $H = \frac{m}{t}$, where $m > 1$. The equations of cosmological dynamics are analyzed for private values of $\omega(T)$ with the following choice of the function F: $F = T + \omega(T)\nabla_\mu T\nabla^\mu T$. It is shown that the energy and pressure of the dark sector, arising due to higher derivatives, weaken over large times. At the early stage of the Universe's evolution, they support power-law inflation, and the dependence of the potential $V(\phi)$ on the scalar field ϕ is justified from a physical point of view.

Keywords: Modified teleparallel gravity, power-law inflation, dark energy.

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Introduction

The modified teleparallel gravity with higher-derivative torsion terms have been introduced into consideration as analog of $f(R,(\nabla R)^2,\Box R)$ gravity [1,2]. It is clear that for such complicated theory as $F(T,(\nabla T)^2,\Box T)$ gravity rather difficult to find exact or approximate solutions without some simplification of the model. Therefore we started from study of truncated $F(T,(\nabla T)^2)$ model where the function F is represented as $F(T,(\nabla T)^2) = F_1(T) + F_2(T,X)$ [3], where $X = \nabla_\mu T \nabla^\mu T$. Further simplifications in this work are the choice of F_1 part corresponding to TEGR as $F(T,(\nabla T)^2) = T + F_2(T,X)$ (where $F_2 = \omega(T)X$) and consideration of power-law expansion of the Universe $a(t) = a_s t^m$, ($a_s = const.$, $m > 1$). The equations of cosmological dynamics are analyzed for private values of $\omega(T)$ with the following choice of the function F: $F = T + \omega(T)\nabla_\mu T \nabla^\mu T$. It is shown that the energy and pressure of the dark sector, arising due to higher derivatives, weaken over large times. At the early stage of the Universe's evolution, they support power-law inflation, and the dependence of the potential $V(\phi)$ on the scalar field ϕ is justified from a physical point of view.

1. Equations of cosmological dynamics

The modified teleparallel gravity with higher-derivative torsion terms, of the form $F(T,(\nabla T)^2,\Box T)$ was constructed in [1]. The action of the model is expressed as follows:

$$S = \frac{1}{2}\int d^4x\, e\, F(T,(\nabla T)^2,\Box T) + S_m(e^A_\rho, \Psi_m), \tag{1}$$

where the light speed $c = 1$, Einstein's gravitational constant $\kappa = 8\pi G = 1$, T is the torsion scalar, $e = det(e^A_\mu) = \sqrt{-g}$ the determinant of the tetrad, $\mu, \nu, \ldots = 0, 1, 2, 3$; $(\nabla T)^2 = \eta^{AB} e^\mu_A e^\nu_B \nabla_\mu T \nabla_\nu T = g^{\mu\nu}\nabla_\mu T \nabla_\nu T$; $\Box T = \nabla_\mu \nabla^\mu T$. S_m is a general matter action comprised of general fields Ψ_m. We are going to start with a truncated version $F(T,(\nabla T)^2)$ in the Friedmann-Robertson-Walker (FRW) metric:

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j, \ (i,j,\ldots = 1,2,3), \tag{2}$$

which arises from the tetrad $e^A_\mu = diag(1, a(t), a(t), a(t))$, where $a = a(t)$ is the scale factor. The equations of cosmological dynamics in the tetrad corresponding to the FRW metric are presented in the following form:

$$3H^2 = \rho_m + \rho_{DE}, \quad -2\dot{H} = \rho_m + p_m + \rho_{DE} + p_{DE} \tag{3}$$

where ρ_m and p_m denote the density and pressure of the matter component. ρ_{DE} and p_{DE} are the effective terms arising from higher-derivatives for truncated model by the following way:

$$\rho_{DE} = -\frac{F}{2} - 6H^2 F_T + 3H^2 - 432\dot{H}H^4 F_X - 144\ddot{H}H^3 F_X - 144\dot{H}H^3 \dot{F}_X, \tag{4}$$

$$p_{DE} = -3H^2 - 2\dot{H} + 2\dot{H}F_T + 3H^2 F_T + 2H\dot{F}_T + 96\ddot{H}H^2\dot{F}_X + 144\dot{H}^2 H\dot{F}_X + 144\dot{H}H^3\dot{F}_X,$$

$$+48\dot{H}H^2\ddot{F}_X + 48\dddot{H}H^2 F_X + 576\dot{H}^2 H^2 F_X + 48\dot{H}^3 F_X + 192\ddot{H}\dot{H}H F_X + 144\ddot{H}H^3 F_X, \tag{5}$$

where $X = (\nabla T)^2$; $T = -6H^2$; $\dot{T} = \frac{dT}{dt} = -12H\dot{H}$; $X = \dot{T}^2 = 144H^2\dot{H}^2$, $F_X = \frac{dF}{dX}$, $F_T = \frac{dF}{dT}$.

By analogy with the investigation of $f(R,(\nabla R)^2)$ [2] we choose the function $F(T,(\nabla T)^2)$ as follows:

$$F = T + \omega(T)X. \tag{6}$$

Such choice means that we kept teleparallel equivalent of GR (TEGR) when kinetic part is absent, i.e. $\omega(T) = 0$. Let us study few specific cases:

A. $F = T + \alpha_1 X$, where $\omega(T) = \alpha_1 = const.$

B. $F = T + \alpha_2 TX$, where $\omega(T) = \alpha_2 T$, $\alpha_2 = const.$

C. $F = T + \frac{\alpha_3}{T}X$, where $\omega(T) = \frac{\alpha_3}{T}$, $\alpha_3 = const.$

For each case, the derivatives of the function $F(T, (\nabla T)^2)$ with respect to T, X and time t are computed as follows:

$$(A): \quad F_T = 1;\ \dot{F}_T = 0;\ F_X = \alpha_1;\ \dot{F}_X = 0;\ \ddot{F}_X = 0, \tag{7}$$

$$(B): \quad F_T = 1 + \alpha_2 X;\ \dot{F}_T = \alpha_2 \dot{X};\ F_X = \alpha_2 T;\ \dot{F}_X = \alpha_2 \dot{T};\ \ddot{F}_X = \alpha_2 \ddot{T}, \tag{8}$$

$$(C): \quad F_T = 1 - \frac{\alpha_3 X}{T^2};\ \dot{F}_T = 2\frac{\alpha_3 X}{T^3}\dot{T} - \frac{\alpha_3}{T^2}\dot{X};\ F_X = \frac{\alpha_3}{T};\ \dot{F}_X = -\frac{\alpha_3}{T^2}\dot{T};\ \ddot{F}_X = \frac{2\alpha_3 \dot{T}^2}{T^3} - \frac{\alpha_3 \ddot{T}}{T^2}. \tag{9}$$

Substituting these values into the field equations (3) with density (4) and pressure (5) of dark energy, we can derive the field equations for these models.

For the case (A), $F = T + \alpha_1 X$ we obtained the following expressions:

$$\rho_{DE} = -72\alpha_1 \dot{H}^2 H^2 - 432\alpha_1 \dot{H} H^4 - 144\alpha_1 \ddot{H} H^3, \tag{10}$$

$$p_{DE} = 48\alpha_1 \dddot{H} H^2 + 576\alpha_1 \dot{H}^2 H^2 + 48\alpha_1 \dot{H}^3 + 192\alpha_1 \ddot{H}\dot{H} H + 144\alpha_1 \ddot{H} H^3. \tag{11}$$

For the case (B), $F = T + \alpha_2 TX$, the expressions for density and pressure are given by:

$$\rho_{DE} = 2592\alpha_2 \dot{H} H^6 + 864\alpha_2 \ddot{H} H^5 + 1296\alpha_2 \dot{H}^2 H^4, \tag{12}$$

$$p_{DE} = -3024\alpha_2 \dot{H}^2 H^4 - 864\alpha_2 \ddot{H} H^5 - 1728\alpha_2 \dot{H}^3 H^2 - 2304\alpha_2 \ddot{H}\dot{H} H^3 - 288\alpha_2 \dddot{H} H^4 - 1728\alpha_2 \dot{H}^2 H^2. \tag{13}$$

Finally, for the case (C), $F = T + \frac{\alpha_3}{T}X$ we have:

$$\rho_{DE} = 72\alpha_3 \dot{H} H^2 - 12\alpha_3 \dot{H}^2 + 24\alpha_3 \ddot{H} H, \tag{14}$$

$$p_{DE} = 48\alpha_3 \dot{H}^2 - 108\alpha_3 \dot{H}^2 H^2 - 8\alpha_3 \dddot{H} H^2 - 24\alpha_3 \ddot{H} H^3. \tag{15}$$

In all three cases, when the coefficients α_1, α_2 and α_3 are equal to zero, we obtain the teleparallel equivalent of general relativity, as ρ_{DE} and p_{DE} are vanishing. The energy and pressure of dark sector are used to describe the later inflation. To study early cosmological inflation it is possible to make transformation to self-interacting scalar field with the following relations

$$\dot{\phi}^2 = \rho_{DE} + p_{DE}, \quad 2V = \rho_{DE} - p_{DE}, \tag{16}$$

where $\phi(t)$ is the scalar field, $V(\phi)$ the potential energy of the field ϕ.

2. Power-Law Inflation

Let us consider a power-law evolution of the scale factor $a(t) = a_s t^m$, where a_s is the constant. Evidently the Hubble parameter takes the form $H = \frac{m}{t}$. Specifically for inflationary expansion we choose m greater than one ($m > 1$). This applies to all three aforementioned cases of the model. For each case, we will find the values of density and pressure of dark sector. In the first case (A), by substituting the Hubble parameter $H = \frac{m}{t}$ into the expressions for the density (10) and pressure (11) of dark sector, we obtain the following values:

$$\rho_{DE} = 72\alpha_1 \frac{m^4}{t^6}(6m - 5), \quad p_{DE} = 144\alpha_1 \frac{m^3}{t^6}(6m - 5). \tag{17}$$

By performing similar transformations using equations (12) and (13) for the second case (B), and equations (14) и (15) for the third case (C), we obtain:

$$\rho_{DE} = -432\alpha_2 \frac{m^6}{t^8}(6m - 7), \quad p_{DE} = -144\alpha_2 \frac{m^5}{t^8}(45m - 56), \tag{18}$$

$$\rho_{DE} = 36\alpha_3 \frac{m^2}{t^4}\left(1-2m\right), \quad p_{DE} = 12\alpha_3 \frac{m}{t^4}\left(4-9m\right). \tag{19}$$

By taking the limit $t \to \infty$ in equations (17), (18) and (19) , we observe a diminishing influence of dark energy on cosmological dynamics. Thus, these models are unable to adequately describe late inflation. Furthermore, examining the equation of state of dark sector given by $w = \frac{p_{DE}}{\rho_{DE}}$, we can see that it is time-independent and has a specific constant value for each case, depending on the expansion exponent m:

$$\text{(A)}: w = \frac{2}{m}, \quad \text{(B)}: w = \frac{3m\left(6m-7\right)}{45m-56}, \quad \text{(C)}: w = \frac{9m-4}{3m(2m^2-1)}. \tag{20}$$

Note, that in the case (B) we have the range $1.166 < m < 1.244$ which leads to negative w.

Now, let us consider existence of the early power-law inflation.

Using the relations for the scalar field and the potential (16) we can find the dependence of the potential on the scalar field.

For the case (A), using (17) we find that this dependence will be cubic:

$$V = \frac{2}{3}\frac{m-2}{\left(m+2\right)\sqrt{2\alpha_1 m^3\left(6m-5\right)\left(m+2\right)}}\phi^3. \tag{21}$$

For the case (B), using (18), we find that this dependence contains ϕ in the power $\frac{3}{8}$ and it will have restrictions on the expansion parameter if :$1 < m \lesssim 1.2$, $\alpha_2 > 0$:

$$V = 144\alpha_2 \frac{m^5\left(3m-7\right)\left(3m-4\right)}{\left(\mp 4\sqrt{-2\alpha m^5\left(9m^2+12m-28\right)}\right)^{\frac{8}{3}}}\phi^{\frac{8}{3}}. \tag{22}$$

For the case (C), using (19) , we find that this dependence is quartic:

$$V = -\frac{13\left(3m^2-6m+2\right)}{576\alpha_3 m\left(3m^2+3m-2\right)}\phi^4. \tag{23}$$

Below are the graphs of the dependencies of the potential V on the scalar field ϕ. The values of m and the coefficients $\alpha_1, \alpha_2, \alpha_3$ from equations (21), (22) и (23) have been selected for clarity, and they are specified in the descriptions accompanying the graphs. Considering specific cases of the model, one

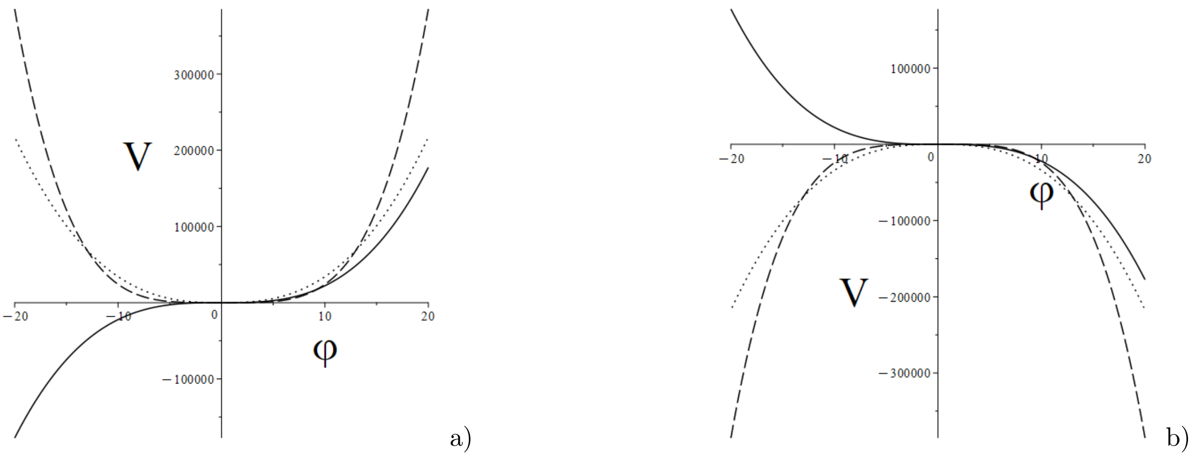

Рис. 1. Graph of the dependencies of the potential V on the scalar field ϕ a) The solid line represents the model $F = T + \alpha_1 X$, with $\alpha_1 = -1000$ and $m = 1,3$; the dashed line is for the model $F = T + \alpha_2 TX$, with $\alpha_2 = 1$ and $m = 1,2$ and the dotted line corresponds to the model $F = T + \frac{\alpha_3}{T}X$, with $\alpha_3 = 0.0001$ and $m = 1,3$ b) The solid line represents the model $F = T + \alpha_1 X$, with $\alpha_1 = 1000$ and $m = 1,3$; the dashed line is for the model $F = T + \alpha_2 TX$, with $\alpha_2 = -1$ and $m = 1,2$ and the dotted line corresponds to the model $F = T + \frac{\alpha_3}{T}X$, with $\alpha_3 = -0.0001$ and $m = 1,3$

can conclude that models (B) and (C) are suitable for describing the behavior of the early universe in power-law inflation, while the model (A) has not minimum (maximum) of the potential and it does not admit chaotic inflation at least.

Conclusion

Our result is as following. Considered power-law models could not effectively describe the later inflation, because of dark energy weaken on large times. As for early inflation, we found that models (B) and (C) have suitable from physical point of view potentials which can support chaotic inflation. Also, we note that power-law evolution of the scale factor does not admit "graceful exit" from inflation, nevertheless this model gives opportunity to find the method of investigation more realistic models, say exponential power-law inflation.

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