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ОПИСАНИЕ ГЛОБАЛЬНОЙ ЭВОЛЮЦИИ ВСЕЛЕННОЙ НА ОСНОВЕ ОБОБЩЕННЫХ РЕШЕНИЙ УРАВНЕНИЙ КОСМОЛОГИЧЕСКОЙ ДИНАМИКИ

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Рассматриваются различные стадии эволюции вселенной для случая гравитации Эйнштейна на основе точных решений уравнений космологической динамики для одного парамера Хаббла в различных режимах со скалярным полем и различными материальными полями. Показано, что полученные решения соответсвуют корректной смене стадий эволюции вселенной. Также показано, что предложенная модель соотвествует наблюдательным ограничениям на значения параметров космологичсеких возмущений. Также обсуждается физическая интерпретация скалярного поля в контексте его фундаментальной и эффективной трактовки.

Ключевые слова: космологические модели, скалярное поле, реликтовые гравитационные волны.

DESCRIPTION OF THE GLOBAL EVOLUTION OF THE UNIVERSE BASED ON GENERALIZED SOLUTIONS OF THE COSMOLOGICAL DYNAMIC EQUATIONS

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The various stages of the universe evolution are considered for the case of Einstein gravity based on exact solutions of the equations of cosmological dynamics for one Hubble parameter in various regimes with a scalar field and different matter fields. It is shown that the obtained solutions correspond to the correct change of the stages of the universe evolution. It is also shown that the proposed model corresponds to observational constraints on the values of the parameters of cosmological perturbations. The physical interpretation of the scalar field is also discussed in the context of its fundamental and effective treatment.

Keywords: cosmological models, scalar field, relic gravitational waves.

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Introduction

Currently, a large number of cosmological models are being considered that describe various stages of the evolution of the universe [1, 2]. Some of the simplest cosmological models are quintessence ones in which a single scalar field describes both stages of the accelerated expansion of the universe [3].

In this paper, we consider a cosmological model that includes all known stages of the evolution of the universe based on a single Hubble parameter. It is shown that the first and second inflations are described by a scalar field only at small and large times. The effective character of the scalar field in the proposed exact solutions of the cosmological dynamic equations is also discussed.

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1. Exact solutions of the cosmological dynamic equations

To analyze a different stages of the universe's evolution we consider the Friedmann universe filled with the scalar field and the additional material field as the ideal barotropic fluid. The pressure p_m and density ρ_m of the additional material field are related by the equation of state $p_m = w_m \rho_m$, similar to the case of a scalar field $p_\phi = w_\phi \rho_\phi$.

We write the action for these models in system of units $8\pi G = c = 1$ as follows [1, 2]

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] + \int d^4x\sqrt{-g}\mathcal{L}_m, \tag{1}$$

where $\mathcal{L}_m$ is the Lagrangian of the additional material field.

In the case where the material field is considered as a barotropic ideal fluid, for the action (1) in a spatially flat Friedmann-Robertson-Walker (FRW) space-time

$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right), \tag{2}$$

one has the following dynamic equations [2]

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_m = \rho_\phi + \rho_m, \tag{3}$$

$$3H^2 + 2\dot{H} = -\frac{1}{2}\dot{\phi}^2 + V(\phi) - p_m = -p_\phi - p_m, \tag{4}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'_\phi = 0, \tag{5}$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0, \tag{6}$$

where an over dot denotes ordinary derivative with respect to cosmic time, Hubble parameter $H = \dot{a}/a$ and $V'_\phi = dV/d\phi$.

State parameter of the additional material component [2]

$$w_m \equiv (m-3)/3, \tag{7}$$

is defined in terms of the constant m.

The evolution of the density and pressure of the additional material field can be defined due to Eq. (6) as follows

$$\rho_m = \frac{\rho_{0m}}{a^m}, \qquad p_m = \left(\frac{m-3}{3}\right)\frac{\rho_{0m}}{a^m}, \tag{8}$$

where ρ_{0m} is the initial density of any additional material field.

We consider exact solutions of dynamic equations (3)–(6) for the Hubble parameter

$$H(t) = -\lambda + h(t) = -\lambda + 2B\coth\left(Bmt\right), \tag{9}$$

in different regimes, where λ and B are a some constants.

For the first inflation with $\rho_m = p_m = 0$ and arbitrary m exact solutions of dynamic equations (3)–(6) are

$$a(t) \sim \exp(-\lambda t)t^{\lambda_1}, \tag{10}$$

$$\phi(t) = \phi_0 \pm \sqrt{2\lambda_1}\ln(\mu t), \tag{11}$$

$$V(\phi) = \frac{9\lambda^2\lambda_1}{3\lambda_1 - 1}\left[1 - \mu\left(\frac{3\lambda_1 - 1}{3\lambda}\right)e^{-\frac{(\phi-\phi_0)}{\sqrt{2\lambda_1}}}\right]^2 - \frac{3\lambda^2}{3\lambda_1 - 1}, \tag{12}$$

where $\lambda_1 = 2/m > 1/3$, ϕ_0 and μ is a some constant.

The dependence of the tensor-to-scalar ratio from the spectral index of the scalar perturbations for this model is [4]

$$r = 4\lambda_1(1-n_S)^2. \tag{13}$$

Under condition $\lambda_1 > 1/3$ for $n_S \simeq 0.97$ taking into account observational constraint on the value of the tensor-to-scalar ratio $r < 0.032$ [4], one has $0.0012 < r < 0.0320$. This expression restricts the possible contribution of relic gravitational waves to the anisotropy and polarization of CMB [1,2].

For kination ($m = 6$), radiation domination ($m = 4$) and matter domination ($m = 3$) stages exact solutions of dynamic equations (3)–(6) are

$$a(t) = (mAB)^{2/m}\, t^{2/m}, \tag{14}$$

$$\phi(t) = \phi_0 + 4\sqrt{\frac{k_m}{(k_m+1)m}}\ln(Bmt), \tag{15}$$

$$V(\phi) = \frac{2k_m B^2(6-m)}{k_m+1}\exp\left(-\sqrt{\frac{(k_m+1)m}{4k_m}}(\phi-\phi_0)\right), \tag{16}$$

$$k_m = \frac{\rho_\phi}{\rho_m} = \frac{p_\phi}{p_m} = \frac{12A^2B^2}{\rho_{0m}} - 1 = const > 0, \tag{17}$$

with the same state parameter for the scalar field and additional material field $w_m = w_\phi$.

For the modern stage of the universe's evolution ($m = 3$ and $k \equiv k_3$) with $|\lambda| \ll h(t)$, exact solutions of dynamic equations (3)–(6) are

$$a(t) = A^{2/3}\sinh^{2/3}(3Bt), \tag{18}$$

$$\phi(t) = \phi_0 \pm 4\sqrt{\frac{k}{3(k+1)}}\operatorname{arctanh}\left(e^{3Bt}\right), \tag{19}$$

$$V(\phi) = \frac{24kB^2}{k+1}\left[\cosh^2\left(\frac{1}{4}\sqrt{\frac{3(k+1)}{k}}(\phi-\phi_0)\right) - \frac{1}{2}\right]^2 + \frac{6B^2(k+2)}{k+1}, \tag{20}$$

where state parameter is

$$w = \frac{p_\phi + p_m}{\rho_\phi + \rho_m} = -\tanh^2(3Bt). \tag{21}$$

For the case $k = 0$ these solutions corresponds to the ΛCDM-model with cosmological constant $V = \Lambda = 12B^2$ and the same scale factor (18). However, we will consider the case $k \neq 0$ corresponding to ϕCDM-model instead of ΛCDM-model.

Also, due to expressions (8) and (18)–(20) relation between energy density of the scalar field and additional material field

$$\frac{\rho_\phi}{\rho_m} = (k+1)\cosh^2(3Bt) - 1. \tag{22}$$

is rapidly increasing function.

Thus, at the large times this model is reduced to the universe filled with a scalar field only with negligible other material fields (dark matter and baryonic matter).

For the second inflation with $\rho_m = p_m = 0$, $|\lambda| \sim h(t)$ and arbitrary m exact solutions of dynamic equations (3)–(6) are

$$a(t) \sim e^{-\lambda t}\sinh^{2/m}(Bmt), \tag{23}$$

$$\phi(t) = \phi_0 \pm \frac{4}{\sqrt{m}}\operatorname{arctanh}\left(e^{Bmt}\right), \tag{24}$$

$$V(\phi) = 8B^2(6-m)\left[\cosh^2\left(\frac{\sqrt{m}(\phi-\phi_0)}{4}\right) - \frac{Bm-6B+3\lambda}{2(m-6)B}\right]^2 + \frac{m\left(2B^2m - 12B^2 + 3\lambda^2\right)}{m-6}. \tag{25}$$

For even longer times in future era, solutions (23)–(25) can be written as follows

$$H = 2B - \lambda = const, \quad \phi = const, \quad V = \Lambda_{\infty} = 3\,(2B - \lambda)^2 = const. \tag{26}$$

Thus, the final stage implies either an infinite expansion or a collapse of the universe filled with a cosmological constant $\Lambda_{\infty} = 3\,(2B - \lambda)^2$.

Conclusion

As one can see, all known stages of the universe's evolution can be described on the basis of exact solutions of cosmological dynamic equations for one Hubble parameter in different regimes. In this case, one scalar field is sufficient to describe both stages of the accelerated expansion of the universe. However, the fact that in the radiation and matter dominated stages the scalar field behaves exactly like the corresponding matter fields leads to the possibility of its effective treatment within the framework of modified gravity theories beyond Einstein gravity [1] to analyze all stages of the evolution of the universe.

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