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**ВЕРИФИЦИРОВАННЫЕ МОДЕЛИ КОСМОЛОГИЧЕСКОЙ ИНФЛЯЦИИ С
ОБОБЩЕННЫМ ПОТЕНЦИАЛОМ СКАЛЯРНОГО ПОЛЯ**Фомин И. В.^{a,1}, Глушков В. Л.^{a,2}, Денцель Е. С.^{a,3}, Манучарян Г. Д.^{a,b,4}^a МГТУ им. Н.Э. Баумана, г. Москва, 105005, Россия.^b ГАИШ МГУ им. М.В. Ломоносова, г. Москва, 119234, Россия.

Рассматриваются модели космологической инфляции с обобщённым потенциалом на основе точных решений уравнений космологической динамики. В данном случае, механизм реализации инфляционной стадии зависит от выбора параметров модели.

Построенные модели удовлетворяют современным наблюдательным ограничениям на параметры космологических возмущений для сверх-планковских скалярных полей. Показано, что в случае гравитации Эйнштейна-Гаусса-Бонне возможно применить предложенный подход для суб-планковских скалярных полей.

Также показано, что при учете неминимальной связи скалярного поля и скаляра Гаусса-Бонне ожидаемый вклад реликтовых гравитационных волн в анизотропию и поляризацию реликтового излучения существенно меньше современных наблюдательных ограничений.

Ключевые слова: Космологическая инфляция, гравитация Эйнштейна-Гаусса-Бонне, скалярное поле, параметры космологических возмущений.

**VERIFIED COSMOLOGICAL INFLATION MODELS WITH GENERALISED
SCALAR FIELD POTENTIAL**Fomin I. V.^{a,1}, Glushkov V. L.^{a,2}, Dentsel E. S.^{a,3}, Manucharyan G. D.^{a,b,4}^a Bauman Moscow State Technical University, Moscow, 105005, Russia.^b Sternberg Astronomical Institute, Moscow State University, Moscow, 119234, Russia.

Cosmological inflation models with a generalized potential based on exact solutions of the cosmological dynamics equations are considered. In this case, the mechanism for implementing the inflationary stage depends on the choice of model parameters.

The constructed models satisfy modern observational constraints on the parameters of cosmological perturbations for super-Planck scalar fields. It is shown that in the case of Einstein-Gauss-Bonnet gravity it is possible to apply the proposed approach for sub-Planck scalar fields.

It is also shown that when taking into account the non-minimal coupling of the scalar field and the Gauss-Bonnet scalar, the expected contribution of relic gravitational waves to the anisotropy and polarization of CMB is significantly less than modern observational constraints.

Keywords: Cosmological inflation, Einstein-Gauss-Bonnet gravity, scalar field, cosmological perturbations parameters.

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Introduction

The cosmological inflation theory is currently the most consistent approach to describe the early Universe evolution and explain the large-scale structure origin [1, 2]. There are various different inflationary scenarios based both on Einstein's gravity and modified theories of gravity as well [1, 2]. The simplest models are based on the single scalar field evolution analysis as a material source of the gravitational field and its perturbations at times close to the Planck time.

In the present paper we consider exact solutions of the cosmological dynamics equations for the generalised effective scalar field potential. Each partial potential corresponds to known physical process during inflationary stage of the early universe's evolution.

1. Inflation with generalized potential based on Einstein gravity

Let us consider in system of units $8\pi G = M_P^{-2} = 1$, the action for inflationary models based on Einstein gravity [1, 2]

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right], \quad (1)$$

where R is the Ricci scalar, $g^{\mu\nu}$ is a space-time metric tensor, ϕ is a scalar field, $V(\phi)$ is a scalar field potential.

In the Freedman-Robertson-Walker (FRW) space-time the cosmological background equations can be written as [2]

$$3H^2 = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad (2)$$

$$\frac{1}{2} \dot{\phi}^2 = -\dot{H}, \quad (3)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'_\phi = 0. \quad (4)$$

We present a new type of exact solutions of these equations:

$$H(t) = \lambda - \frac{\beta\phi_0^2}{4} e^{-2\mu} \left[\exp(\beta t + 2e^{\beta t}) - \frac{1}{2} \exp(2e^{\beta t}) \right], \quad (5)$$

$$\phi(t) = \phi_0 \exp(\exp(\beta t) - \mu), \quad (6)$$

$$V(\phi) = \sum_{i=1}^5 V_i(\phi), \quad (7)$$

where scalar field potential is the sum of following components:

$$V_1(\phi) = \frac{3}{8} \beta^2 \left[\frac{\alpha}{4} \left(\mu - \frac{1}{2} \right) + \frac{1}{2} \mu (\mu - 1) + \eta + \frac{1}{8} \right] \phi^4 - \frac{1}{2} \beta \left[3\lambda \left(\mu - \frac{1}{2} \right) + \beta\mu(\mu + \omega) + \frac{3}{2} \omega\lambda \right] \phi^2 + k_1 \lambda^2, \quad (8)$$

$$V_2(\phi) = -\frac{1}{2} \beta^2 \phi^2 \ln^2 \left(\frac{\phi}{\phi_0} \right), \quad (9)$$

$$V_3(\phi) = \frac{3}{16} \beta^2 \phi^4 \left[\ln^2 \left(\frac{\phi}{\phi_0} \right) - \eta \right], \quad (10)$$

$$V_4(\phi) = \beta \left(\beta\mu + \frac{3}{2} \lambda \right) \phi^2 \left[-\frac{\omega}{2} + \ln \left(\frac{\phi}{\phi_0} \right) \right] + k_2 \lambda^2, \quad (11)$$

$$V_5(\phi) = \frac{3}{8} \beta^2 \left(\mu - \frac{1}{2} \right) \phi^4 \left[\ln \left(\frac{\phi}{\phi_0} \right) - \frac{\alpha}{4} \right] + k_3 \lambda^2, \quad (12)$$

and ϕ_0 , β , μ , α , ω and η are some positive constants.

We note that each of these potentials corresponds to the well-known inflationary model [3]. However, for generalized potential (7) we can consider all this types on inflationary processes based on a single class of exact solutions of the cosmological dynamic equations.

We also note that this cosmological inflation model satisfies the observational constraints on the parameters of cosmological perturbations [4]

$$\mathcal{P}_S = 2.1 \times 10^{-9}, \quad n_S = 0.9663 \pm 0.004, \quad r < 0.032, \quad (13)$$

for e-folds number $\Delta N = N_e - N_* = 62$ and scalar field values between end of inflation (for $t = t_{end}$) and crossing of the Hubble radius (for $t = t_*$) is $\Delta\phi = |\phi_{end} - \phi_*| \simeq 10$. As one can see, the scalar field takes super-Planck values during inflation for the Einstein gravity case.

2. Inflation with generalized potential based on Einstein-Gauss-Bonnet gravity

To reduce the scalar field values to sub-Planck ones we consider the influence of the non-minimal coupling of the scalar field and the Gauss-Bonnet scalar on the cosmological parameters.

The action of the inflationary models based on Einstein-Gauss-Bonnet (EGB) gravity is [5]

$$S_{GB} = \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi_{GB} \partial_\nu \phi_{GB} - V_{GB}(\phi_{GB}) - \frac{1}{2} \xi(\phi_{GB}) R_{GB}^2 \right], \quad (14)$$

where $R_{GB}^2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the Gauss-Bonnet term, and $\xi = \xi(\phi_{GB})$ is the non-minimal coupling function.

For the special choice of the coupling function [5]

$$\xi(\phi_{GB}(t)) = \frac{\alpha_{GB}}{4H^2(\phi_{GB}(t))} + \xi_0, \quad (15)$$

in FRW space-time under slow-roll conditions one has the same cosmological parameters as for the case of Einstein gravity except the scalar field and the tensor-to-scalar ratio:

$$\phi_{GB} = \phi \sqrt{1 - \alpha_{GB}}, \quad (16)$$

$$r_{GB} = r(1 - \alpha_{GB}), \quad (17)$$

where α_{GB} is the coupling constant.

Taking into account relations (16)–(17), we can reduce the scalar field to the sub-Planck values $\Delta\phi_{GB} \leq 1$ for $\alpha_{GB} \approx 0.998$ and $r_{GB} \leq 7 \times 10^{-5}$.

We note that cosmological inflation models based on the generalized potential (7), taking into account the non-minimal coupling of the scalar field and the Gauss-Bonnet term, predict a significantly smaller contribution of relic gravitational waves to the anisotropy and polarization of CMB than in the case of Einstein gravity.

Conclusion

In this paper we consider cosmological inflation models based on the generalised scalar field potential containing as the effective potentials components corresponding to the various physical effects realization characteristic of theoretical models of the early universe evolution. Note that the considered generalised potential is obtained on the basis of cosmological dynamics equations exact solutions in contrast to the standard way of analyzing inflationary models on the slow-roll approximation basis.

It is shown that in contrast to inflation based on Einstein gravity with the Higgs potential, which does not satisfy observational constraints on the parameters values of cosmological perturbations, inflationary models based on the proposed generalised potential satisfy these constraints for the case of super-Planckian scales of the scalar field evolution.

Also, considering the influence of the non-minimal coupling of the scalar field and Gauss-Bonnet scalar on the parameters of the considered cosmological model it was shown that unlike the Einstein

gravity case the proposed models of cosmological inflation can be considered as verified by observational data for sub-Planckian fields. It should also be noted that taking into account the influence of the non-minimal coupling of the scalar field and the Gauss-Bonnet scalar reduces the expected value of the contribution of relic gravitational waves compared to scalar perturbations to the values $r_{\text{GB}} \leq 6.95 \times 10^{-5}$, which implies a significantly smaller amplitude of relic gravitational waves than the present observational constraints. We also note that inflationary models based on the Higgs field non-minimally coupled to scalar curvature give a different estimate of the tensor-scalar relation. As a the correctness check of these models of cosmological inflation, it is necessary to consider the direct registration of relic gravitational waves, which is possible only at a substantial development methods of their detection.

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