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О КОВАРИАНТНЫХ КУБИЧНЫХ ВЕРШИНАХ ДЛЯ НЕПРИВОДИМЫХ ПОЛЕЙ ВЫСШИХ ЦЕЛЫХ И ПОЛУЦЕЛЫХ СПИНОВ НА ПЛОСКОМ ФОНЕ

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Поля высших спинов (при $s \geq 2$) входят в спектр суперструнной теории, описывающей гравитационное взаимодействие на ранней стадии эволюции Вселенной и являются естественными кандидатами для описания Темной Материи. В сообщении предложена процедура построения Пуанкаре-ковариантных лагранжевых кубичных вершин взаимодействия (ранее известных лишь в формализме светового конуса) безмассовых неприводимых полей высших целых и полуцелых спинов на пространстве-времени Минковского произвольной размерности в рамках подхода Бекки-Рюэ-Стора-Тютина (БРСТ) с неполным БРСТ оператором. Рассмотрение основано на лагранжевых формулировках второго порядка для свободных полностью симметричных неприводимых тензорных и спин-тензорных полей с абелевыми калибровочными симметриями, которые деформируются с сохранением числа физических степеней свободы до кубично взаимодействующей теории с неабелевой калибровочной алгеброй. Общее кубичное взаимодействие для полей спиральностей $n_1+1/2$, $n_2+1/2$, s_3, как и в Стандартной Модели, содержит два типа вершин, являющихся БРСТ-замкнутыми решениями производящих уравнений, удовлетворяющими гамма-бесследовым, бесследовым связям и спиновым условиям.

Ключевые слова: поля высших спинов, БРСТ подход, кубичная вершина, лагранжева формулировка.

ON COVARIANT CUBIC VERTICES FOR IRREDUCIBLE HS FIELDS WITH INTEGER AND HALF-INTEGER SPINS ON FLAT BACKGROUNDS

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Higher-spin fields (for $s \geq 2$) are included into the spectrum of superstring theory, which describes gravitational interaction at an early stage of the Universe evolution and seem to be by natural candidates for describing Dark Matter. The note proposes a procedure for constructing Poincare-covariant Lagrangian cubic interaction vertices (previously known only in the light-cone formalism) for massless irreducible fields of higher integer and half-integer spins on Minkowski space-time of arbitrary dimension within the Becchi-Rouet-Stora-Tyutin (BRST) approach with an incomplete BRST operator. The consideration is based on the second order Lagrangian formulations for free totally-symmetric irreducible tensor and spin-tensor fields with Abelian gauge symmetries, which are deformed with the preservation of the number of physical degrees of freedom to a cubically interacting theory with non-Abelian gauge algebra. General cubic interaction for the fields with helicities $n_1+1/2$, $n_2+1/2$, s_3, as in the Standard Model, contains two types of vertices to be BRST closed solutions of the generating equations, satisfying as well to gamma-traceless? traceless constraints and to the spin conditions.

Keywords: higher-spin fields, BRST approach, cubic vertex, Lagrangian formulation.

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Introduction

Possibilities to formulate consistent quantum gravity make an interest to the higher spin (HS) field theory more and more intense. Simultaneously, it may provide to discover both the physics beyond the Standard Model, with help of new elementary particles of matter and carriers of higher spin interactions and the massive HS fields appear by natural candidates to describe the problem of Dark Matter [1], [2], within. e.g. modified gravity concept [3], [4], [5] (alternatively to models of vector and sterile neutrinos [6]). These expectations are based on connection of HS fields with (Super(string Field Theory on constant curvature spaces [7], operating with infinite sets of massive and massless fields of (half)integer spins (for references and review see, e.g. [8], [9], [10]).

All these points are closely related with general problems in modern theoretical high-energy physics consisting in the construction of interactions with HS fields within gauge-invariant Lagrangian formulations as the most appropriate objects for quantization and calculating S-matrix elements. Whereas the cubic and quartic vertices for various HS fields with integer spins have been studied by many authors in the framework of different approaches (see, e.g., the papers for cubic [11], [12], [13], [14], [15], [16], [17], [18], [19] and for quartic vertices [20]), this problem for interacting irreducible HS fields with half-integer spins was considered only in terms of physical degrees of freedom in the light cone formalism in [21].

The aim of the note is to find the general covariant Lagrangian cubic vertices for massless totally-symmetric HS fields with half-integer (fermionic) and integer (bosonic) helicities in $\mathbb{R}^{1,d-1}$ within BRST approach with incomplete (due to absence in it of the algebraic constraints) BRST operator elaborated for bosonic in [22] and for fermionic HS fields in [23]. BRST approach presents the realization of the AKSZ model [24] and was born due to BRST symmetry discovered in [25], [26] for the aims of Lagrangian [27] and then for the canonical quantization [28], [29] of the dynamical systems subject to constraints. The definitions and notations from [15], [16] are used for a metric tensor $\eta_{\mu\nu} = diag(+,-,...,-)$, gamma-matrices γ^μ: $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ and $\epsilon(F)$, $gh_H(F)$, $(s)_3$, for the values of Grassmann parity and ghost number of a quantity F, for the triple $(n_1 + 1/2, n_2 + 1/2, s_3)$.

1. Cubic vertices for constrained bosonic and fermionic higher-spin fields

Free massless particles with integer s and half-integer $n + 1/2$ helicities can be described with use of $\mathbb{R}$-valued totally symmetric tensor field $\phi_{\mu_1\ldots\mu_s}(x) \equiv \phi_{\mu(s)}$ and spin-tensor $\psi_{A\mu_1\ldots\mu_n}(x) \equiv \psi_{\mu(n)}$ with suppressed Dirac index $A = 1, ..., 2^{[d/2]}$ subject to the respective wave equations equations

$$\big(\partial^\nu\partial_\nu,\, \partial^{\mu_1},\, \eta^{\mu_1\mu_2}\big)\phi_{\mu(s)} = \vec{0} \quad\Longleftrightarrow\quad \big(l_0,\, l_1,\, l_{11}, g_0 - d/2 - s\big)|\phi\rangle = (0,0,0,s)|\phi\rangle, \tag{1}$$

$$\big(\gamma^\nu\partial_\nu,\, \gamma^{\mu_1}\big)\psi_{\mu(n)} = \vec{0} \quad\Longleftrightarrow\quad \big(t_0,\, t_1, g_0^f - d/2\big)|\psi\rangle = (0,0,n))|\psi\rangle. \tag{2}$$

The vector $|\phi\rangle$, ($|\psi\rangle$ for half-integer spin) and the operators $\{l_0,\, l_1,\, l_{11}, g_0\}$; ($\{t_0,\, l_1,\, g_0^f\}$) here are defined in the Fock space $\mathcal{H}$ ($\mathcal{H}^f$) with the Grassmann-even oscillators a_μ, a_ν^+, ($[a_\mu, a_\nu^+] = -\eta_{\mu\nu}$):

$$|\phi\rangle = \sum_{s\geq 0} \frac{\imath^s}{s!}\phi^{\mu(s)}\prod_{i=1}^{s} a^+_{\mu_i}|0\rangle, \quad \big(l_0,\, l_1,\, l_{11}, g_0\big) = \big(\partial^\nu\partial_\nu,\, -\imath a^\nu\partial_\nu,\, \frac{1}{2}a^\mu a_\mu, -\frac{1}{2}\{a^+_\mu,\, a^\mu\}\big), \tag{3}$$

$$|\psi\rangle = \sum_{n\geq 0} \frac{\imath^n}{n!}\psi^{\mu(n)}\prod_{i=1}^{n} a^+_{\mu_i}|0\rangle, \quad \big(t_0,\, t_1, g_0^f\big) = \big(\imath\tilde{\gamma}^\nu\partial_\nu,\, a^\nu\tilde{\gamma}_\nu, -\frac{1}{2}\{a^+_\mu,\, a^\mu\}\big). \tag{4}$$

In (4) it is introduced Grassmann-odd gamma-matrices [30], [23]: $\tilde{\gamma}^\nu = \tilde{\gamma}\gamma^\nu$, $\tilde{\gamma}^2 = -1$, $\{\tilde{\gamma}, \gamma^\nu\} = 0$ (to provide fermionic character for t_0, t_1). The difference among two sets of operators above and vectors $|\phi\rangle$ (scalar), $|\psi\rangle$ (Dirac spinor) is in the matrix-valued structure of the latter. The interacting Lagrangian formulation (LF) within BRST approach with total: $Q_c^{tot} = (\sum_{i=1}^2 Q^{(i)}_{F|c} + Q^{(3)}_c \mathbf{1}_{2^{[d/2]}})$, incomplete BRST operator in cubic approximation is given with accuracy up to linear in deformation parameter g, for 3-copies of massless HS fields: two fermionic $\psi^{(i)}_{\mu(n_i)}$, $i = 1, 2$, and one bosonic $\phi^{(3)}_{\mu(s_3)}$, including in respective vectors $|\psi^{(i)}\rangle \in \mathcal{H}^{(i)}_f$, $\phi^{(3)}\rangle \in \mathcal{H}^{(3)}$ of helicities $(s)_3$, with own set of oscillators $a^{(ki)}_\mu, a^{+(k)}_\nu$, $k=1,2,3$ (with conventions $n_3 \equiv s_3$ and $[i+3 \simeq i]$)

$$S^{(s)_3}_{[1]|c}[(\chi)_3] \ = \ \sum_{i=1}^{2}\mathcal{S}^{(2)n_i}_{0|c}[\chi^{(i)}] + \mathcal{S}^{s_3}_{0|c}[\chi^{(3)}_B] + +g\int\prod_{k=1}^{3} d\eta^{(e)}_0\Big({}_{n_e}\langle\chi^{(e)}\big|V^{(3)}\rangle_{(s)_3} + h.c.\Big), \tag{5}$$

$$\delta_{[1]}|\chi^{(i)}\rangle_{n_i} = Q^{(i)}_{F|c}|\Lambda^{(i)}\rangle_{n_i} - g\int\prod_{e=1}^{2} d\eta^{(i+e)}_0\Big({}_{n_{i+1}}\langle\Lambda^{(i+1)}\big|{}_{n_{i+2}}\langle\chi^{(i+2)}\big| + (1\leftrightarrow 2)\Big)\big|\widetilde{V}^{(3)}\rangle_{(s)_3}, \tag{6}$$

$$\delta_{[1]}|\chi^{(3)}_B\rangle_{s_3} = Q^{(3)}_c|\Lambda^{(3)}\rangle_{s_3} - g\int\prod_{e=1}^{2} d\eta^{(e)}_0\Big({}_{n_1}\langle\Lambda^{(1)}\big|{}_{n_2}\langle\chi^{(2)}\big| + (1\leftrightarrow 2)\Big)\big|\widehat{V}^{(3)}\rangle_{(s)_3} \tag{7}$$

(for $(\epsilon, gh_H)[Q^{(3)}_c, Q^{(i)}_{F|c}]$=$(1,1)$; $(\epsilon, gh_H)\chi^{(k)}$=$(1-\delta_{k3},0)$; $(\epsilon, gh_H)\Lambda^{(k)} = (\delta_{k3},-1)$). These relations determine the irreducible gauge theory with non-Abelian gauge transformations (for $SU(2)$ gauge group, $3 = 2^2-1$) for the field vectors $|\chi^{(k)}\rangle_{s_k}$ (including the sets of auxiliary fields into joint configuration space), with gauge parameters $|\Lambda^{(k)}\rangle_{s_k}$ from total Hilbert space $\mathcal{H}^{tot}_c = \oplus_i\mathcal{H}^{(i)}_{tot|f}\oplus\mathcal{H}^{(3)}_{tot}$ for $\mathcal{H}_{tot|f}f^{(i)}$, related to i-th copy of fermionic and $\mathcal{H}^{(3)}_{tot}$ for bosonic fields. They, in turn, depend on pairs of ghost oscillators, Grassmann-even: $q^{(i)}_0, p^{(i)}_0$; and Grassmann-odd: $\eta^{(k)}_0, \mathcal{P}^{(k)}_0$; $\eta^{(k)}_1, \mathcal{P}^{(k)+}_1$; $\eta^{(k)+}_1, \mathcal{P}^{(k)}_1$ with non-trivial commutation relations:

$$[q^{(i)}_0, p^{(i)}_0] = \{\eta^{(k)}_0, \mathcal{P}^{(k)}_0\} = i, \quad \{\eta^{(k)}_1, \mathcal{P}^{(k)+}_1\} = 1, \ \ \texttt{for}, \quad gh_H(q_0, \eta_0, \eta^+_1) = -gh_H(p_0, \mathcal{P}_0, \mathcal{P}^+_1) = 1$$

according to BRST approaches with incomplete nilpotent BRST operators $Q^{(3)}_c$, $Q^{(i)}_{F|c}$ [23]

$$Q^{(3)}_c = \big(\eta_0 l_0 + \eta^+_1 l_1 + \eta_1 l^+{}_1 + \imath\eta^+_1\eta_1\mathcal{P}_0\big)^{(3)};\ Q^{(i)}_{F|c} = \big(q_0 t_0 + \eta_0 l_0 + \eta^+_1 l_1 + \eta_1 l^+{}_1 + \imath[\eta^+_1\eta_1 - q^2_0]\mathcal{P}_0\big)^{(i)}. \tag{8}$$

Poincare irreducibility of the initial fields $\psi^{(i)}_{\mu(n_i)}$, $i=1,2$, and $\phi^{(3)}_{\mu(s_3)}$ is provided by imposing off-shell BRST-extended γ-traceless $\mathcal{T}^{(i)}_1$, traceless $\mathcal{L}^{(3)}_{11}$ constraints with respective spin constraints $\sigma^{(k)}_c$ both on the field vectors $|\chi^{(k)}\rangle$, gauge parameters $|\Lambda^{(k)}\rangle$ and on the three-vectors of $\big|V^{(3)}\rangle_{(s)_3}$, $\big|\widetilde{V}^{(3)}\rangle_{(s)_3}$, $\big|\widehat{V}^{(3)}\rangle_{(s)_3}$. The actions for free HS fields with s_k included in $|\chi^{(k)}\rangle_{s_k} = |\phi^{(k)}\rangle_{s_k} + \mathcal{O}(\eta)$ in (5) and theirs equations of motions, gauge transformations (6), (7) for $g=0$, off-shell constraints have the form (with respective inner products $\langle\psi|\chi\rangle = \int d^dx\psi^*(x)\chi(x)$ [23]), first, for $\phi^{(3)}_{\mu(s_3)}$:

$$\mathcal{S}^{s_3}_{0|c}[\chi^{(3)}_B] = \int d\eta^{(3)}_0\,{}_{s_3}\langle\chi^{(3)}_B|Q^{(3)}_c|\chi^{(3)}_B\rangle_{s_3},\ \ Q^{(3)}_c|\chi^{(3)}_B\rangle_{s_3} = 0; \delta_0|\chi^{(3)}_B\rangle_{s_3} = Q^{(3)}_c|\Lambda_c\rangle_{s_3}, \tag{9}$$

$$\mathcal{L}^{(3)}_{11}\Big(|\chi^{(3)}_B\rangle, |\Lambda^{(3)}\rangle\Big) = \Big(1/2a^{(3)\mu}a^{(3)}_\mu + \eta^{(3)}_1\mathrm{P}^{(3)}_1\Big)\Big(|\chi^{(3)}_B\rangle, |\Lambda^{(3)}\rangle\Big) = \vec{0}, \tag{10}$$

$$\big(|\chi^{(3)}_B\rangle, |\Lambda^{(3)}\rangle\big)_{s_3} = \Big(|\phi^{(3)}\rangle_{s_3} - \mathcal{P}^{(3)+}_1\big\{\eta^{(3)}_0|\phi^{(3)}_1\rangle_{s_3-1} + \eta^{(3)+}_1|\phi^{(3)}_2\rangle_{s_3-2}\big\}, \mathcal{P}^{(3)+}_1|\xi^{(3)}\rangle_{s_3-1}\Big) \tag{11}$$

(for $\int d\eta^{(3)}_0 \equiv \overrightarrow{\partial}_{\eta^{(3)}_0}$); then, in the second order approximation for spin-tensor fields $\psi^{(i)}_{\mu(n_i)}$

$$\mathcal{S}^{(2)n_i}_{0|c}[\chi^{(i)}] = \int d\eta^{(i)}_0\,{}_{n_i}\langle\chi^{(i)}\big|Q^{(i)}_{F|c}\big|\chi^{(i)}\rangle_{s_i},\ \ Q^{(i)}_{F|c}\big|\chi^{(i)}\rangle_{n_i} = 0; \delta_0|\chi^{(i)}\rangle_{n_i} = Q^{(i)}_{F|c}\big|\Lambda^{(i)}\rangle_{n_i}, \tag{12}$$

$$\mathcal{T}^{(i)}_1\Big(|\chi^{(i)}\rangle, |\Lambda^{(i)}\rangle\Big) = \Big(\gamma^\mu a^{(i)}_\mu - \imath\eta^{(i)}_1 p^{(i)}_0 - 2q^{(i)}_0\mathcal{P}^{(i)}_1\Big)\Big(|\chi^{(i)}\rangle, |\Lambda^{(i)}\rangle\Big) = \vec{0}, \tag{13}$$

$$|\chi^{(i)}\rangle_{n_i} = |\psi^{(i)}\rangle_{n_i} + \mathcal{P}^{(i)+}_1\big(q^{(i)}_0\tilde{\gamma}|\chi^{(i)}_1\rangle_{n_i-1} - \eta^{(i)+}_1|\psi^{(i)}_2\rangle_{n_i-2}\big),\ |\Lambda^{(i)}\rangle_{n_i} = \mathcal{P}^{(i)+}_1|\xi^{(i)}\rangle_{n_i-1},. \tag{14}$$

Each copy of fields and gauge parameters is the proper eigen-vectors for the spin constraints with proper spin numbers n_i, s_3 with incomplete spin operators $\sigma^{(k)}_c$, $k=1,2,3$:

$$\left[\sigma^{(k)}_c - \frac{d-2}{2}\right]\Big(\big|\chi^{(k)}\rangle_{n_k}, \big|\Lambda^{(k)}\rangle_{n_k}\Big) = 0,\ \sigma^{(k)}_c = g^{(k)}_0 + \eta^{(k)+}_1\mathcal{P}^{(k)}_1 - \eta^{(k)}_1\mathcal{P}^{(k)+}_1. \tag{15}$$

The three sets of operators $\{Q^{(i)}_{F|c}, \mathcal{T}^{(i)}_1, \mathcal{L}^{(i)}_{11}, \sigma^{(i)}_c\}$ for $i=1,2$ and $\{Q^{(3)}_c, \mathcal{L}^{(3)}_1, \sigma^{(3)}_c\}$ compose closed superalgebras. Note, the triplet LFs for free HS fields (9), (12) are equivalent for LFs with complete

BRST operators for the same HS fields [23] and reduces after resolution of the off-shell constraints to Fronsdal LF [31] for single field $\phi^{(3)}_{\mu(s_3)}$ and for Fang-Fronsdal LF [32] for $\psi^{(i)}_{\mu(n_i)}$.

The Noether identities for the cubic deformation of the fields $(\psi^{(1)}\psi^{(2)}\phi^{(3)})$ of LF for the particles $(0,s_k)$, $k=1,2,3$ in the first degree in deformation (coupling) constant

$$g^1: \qquad \delta_0 S_{1|(s)_3} + \delta_1\Big(\textstyle\sum_{i=1}^2 \mathcal{S}^{(2)n_i}_{0|c}[\chi^{(i)}] + \mathcal{S}^{s_3}_{0|c}[\chi^{(3)}]\Big) = 0, \tag{16}$$

transforms to the system of equations, which for x-local coinciding unknowns $\widetilde{V}^{(3)} = \widehat{V}^{(3)} = V^{(3)}$

$$\big|V^{(3)}\rangle_{(s)_3}(x) = \prod_{k=1}^{3}\delta^{(d)}\big(x-x_k\big)V^{(3)}\prod_{j=1}^{3}\eta_0^{(j)}|0\rangle, \quad |0\rangle \equiv \otimes_{k=1}^3|0\rangle^e, \tag{17}$$

has universal form

$$\Big(Q_c^{tot},\ \mathcal{T}_1^{(i)},\ \mathcal{L}_{11}^{(3)},\ \sigma_c^{(k)} - \frac{d-2}{2}\Big)\big|V^{(3)}\rangle_{(s)_3} = (\vec{0}, n_k), \quad Q_c^{tot} = \sum_{i=1}^{2} Q^{(i)}_{F|c} + Q^{(3)}_c\cdot 1_{2^{[d/2]}} \tag{18}$$

Thus, the BRST-closed, traceless and gamma-traceless vertex should be composed from $a^{(k)+}_\mu, \eta_1^{(k)+}, \mathcal{P}_1^{(k)+}, \mathcal{P}_0^{(k)}$, $q_0^{(i)}, p_0^{(i)}$, $i=1,2$ of spin $(n_1+1/2, n_2+1/2, s_3)$.

The solution of the equations (18) which covariantizes the light-cone cubic vertex [21] has two types of the vertices (K-, and F-vertices) and are constructed from the Q^{tot}_c BRST-closed polynomials of the first order ($L^{(k)}$) in oscillators $a^{(i)+}_{mu}, \eta^{(i)+}$) and in momenta $p^{(k)}_{mu} = -\imath\partial^{(k)}_{mu}$ and (Z) of the third order in oscillators and the first order in $p^{(i)}_{mu}$. Z and $L^{(k)}$ provide respectively Yang-Mills type (according to correspondence: $L^{(1)}\overline{\psi}^{(1)}_{\mu(n_1)}\psi^{(2)}_{\nu(n_2)}\phi^{(3)}_{\rho(s_3)} \mapsto \overline{\psi}^{(1)}_{\mu(n_1-1)\mu}\partial^\mu\psi^{(2)}_{\nu(n_2)}\phi^{(3)}_{\rho(s_3)}$) and gravitational type interactions ((according to: $Z\overline{\psi}^{(1)}_{\mu(n_1)}\psi^{(2)}_{\nu(n_2)}\phi^{(3)}_{\rho(s_3)} \mapsto \overline{\psi}^{(1)}_{\mu(n_1-1)\mu}\partial^\rho\psi^{(2)}_{\nu(n_2-1)}{}^\mu\phi^{(3)}_{\rho(s_3-1)\rho} + cycle(1,2,3)$) among the fields and determined as

$$L^{(k)} \ = \ (p_\mu^{(k+1)} - p_\mu^{(k+2)})a^{(k)\mu+} - \imath\big(\mathcal{P}_0^{(k+1)} - \mathcal{P}_0^{(k+2)}\big)\eta_1^{(k)+}, \tag{19}$$

$$Z = L_{11}^{(12)+}L^{(3)} + cycle(1,2,3),\ L_{11}^{(kk+1)+} = \frac{1}{2}\Big(a^{(k)\mu+}a^{(k+1)+}_\mu - \mathcal{P}_1^{(k)+}\eta_1^{(k+1)+}\Big) + (k \longleftrightarrow k+1) \tag{20}$$

Peculiarity of the vertices with half-integer spin is in the presence of the BRST-closed extended Dirac operator $K^{(12)}$ and operator F

$$\mathbf{K^{(12)}} \ = \ \tilde{\gamma}^\mu\widehat{p}^{(3)}_\mu - \imath q_0^{(1)}\big[\mathcal{P}_0^{(3)} - \mathcal{P}_0^{(2)} - 3\mathcal{P}_0^{(1)}\big] + \imath q_0^{(2)}\big[\mathcal{P}_0^{(3)} - \mathcal{P}_0^{(1)} - 3\mathcal{P}_0^{(2)}\big], \tag{21}$$

$$\mathbf{F} = \Big(L^{(3)} - \frac{1}{2}\big[K^{(12)},\,\widehat{T}_1^{(3)+}\big]_s\Big), \text{ where} \tag{22}$$

$$\widehat{T}_1^{(3)+} \ = \ \Big(1 - \imath\big[q_0^{(2)}p_0^{(1)} + q_0^{(2)}p_0^{(1)}\big]\Big)\tilde{\gamma}^\mu a^{+(3)}_\mu + \imath\big(p_0^{(1)} + p_0^{(2)}\big)\eta_1^{(3)+} - 2\mathcal{P}_1^{(3)+}\big(q_0^{(1)} + q_0^{(2)}\big).$$

Thus, the parity-invariant solution (given by Figure 1) takes the form for cubic K-, and F-vertices

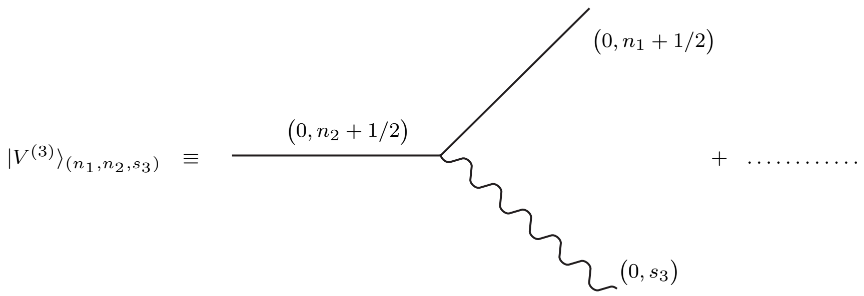

Fig. 1. Interaction vertex $|V^{(3)}\rangle_{(s)_3}$ for massless bosonic $\phi^{(3)}_{\mu(s_3)}$ and two fermionic fields $\phi^{(i)}_{\mu(n_i)}$ of helicities s_3, n_i, $i=1,2$. The terms in "..." correspond to the auxiliary fields from $|\chi^{(k)}\rangle_{s_k}$ before constraints resolution

$$|V_K^{M(3)}\rangle_{(s)_3} = \mathbf{K^{(12)}} \sum_{k=s-2s_{\min}+1}^{s+1} Z^{\frac{1}{2}(s-k+1)} \prod_{i=1}^{3} (L^{(i)})^{n_i - \frac{1}{2}(s-k+1)}|0\rangle, \ s = \sum_1^2 n_i + s_3; \quad (23)$$

$$|V_F^{M(3)}\rangle_{(s)_3} = \mathbf{F} \sum_{k=s-2(\min(n_1,n_2,s_3-1))-1}^{s} Z^{\frac{1}{2}(s-k)} \prod_{i=1}^{3} (L^{(i)})^{n_i-\delta_{i3}-\frac{1}{2}(s-k)}|0\rangle. \quad (24)$$

The vertices above are appropriate for cubic deformations of reducible Poincare group representations with integer and half-integer helicities, whereas for irreducible representations one should to impose additional traceless and gamma-traceless constraints (18) on the vertices [33]. Thus, the each vertex contains k-parameter family of vertices with fixed value of derivatives. For the case of $d = 4$ the number of different vertices is reduced. For $s_3 = s_{min}$ there are two terms with $k = s - 2s_{min} + 1; s + 1$ and one with $k = s + 1$ for $s_3 > s_{min}$ in the **K**-vertex without any parity-invariant **F**-vertex. Whereas for $s_3 > s_{min}$ there is only one **F**-vertex with $k = s - 2s_{min}$. Obtaining the interacting LF (5)–(7) in terms of only initial (spin)-tensors is simply reduced to calculation of the oscillators pairings.

Conclusion

The procedure for BRST approach with incomplete BRST operator to consistently construct covariant local cubic vertices for 3 copies of massless interacting HS fields: one with integer and two fields with half-integer spins on $d$dimensional flat backgrounds with preservation the Poincare group irreps for deformed gauge theory is developed. The parity invariant solution for the generating equations for cubic vertices are firstly found in the covariant form. They contains two types of three-vectors as the generating functions for the cubic vertex in the second order approximation for the LFs for fermionic fields. Of course, not all the vertices among K- and F-vertices will survive under requirement of gauge invariance of the cubically deformed LF in all orders in deformation constant g. For the spin $s_3 = 2$ the obtained solutions open the door for study the interactions of HS matter particles with gravity in covariant form. The problems to get covariant cubic vertices both for these kind of fields in the first-order LFs for the fermionic fields and for massive interacting HS fields also compose the current task's list.

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