УДК 539.12

ПОДХОД ВИНЕРА–БРИОСКИ К РЕЛЯТИВИСТСКОЙ КВАНТОВОЙ МЕХАНИКЕ

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Обсуждаются применения солитонных решений нелинейных уравнений для описания структуры элементарных частиц и обоснования квантовой механики.

Ключевые слова: солитоны, квантовая механика, тождество Бриоски.

WIENER–BRIOSCHI APPROACH TO RELATIVISTIC QUANTUM MECHANICS

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The applications of soliton solutions to nonlinear equations for describing particles structure and substantiating quantum mechanics are discussed.

Keywords: solitons, quantum mechanics, Brioschi identity.

PACS: 11.30 Qc, 04.20.-q.
DOI: 10.17238/issn2226-8812.2025.4.106-109

Introduction

As well known, Einstein suggested a very radical method for solving the divergency problem in quantum field theory– not to consider point-like particles. To this end, he supposed that all elementary particles (such as electrons and photons) should be represented as clots of some universal "unitary field", so-called solitons, that is regular solutions to some nonlinear field equations. In other words, Einstein formulated a tremendous program of geometrizing physics [1]. However, the physical origin of this fundamental field was unknown.

1. Brioschi identity. Topological solitons

Moreover, solitons in future unitary field theory of particles should be stable due to stability of real electrons and protons as constructive elements of atoms. Historically, solution of this stability enigma appears to be connected with the remarkable Euler problem of n squares [2]:

Given n real numbers a_i, $i = \overline{1,n}$, it is necessary to find new n numbers c_i, $i = \overline{1,n}$, considered as bilinear combinations of a_i, with the following relation being satisfied:

$$\left(\sum_{i=1}^{n} a_i^2\right)^2 = \sum_{i=1}^{n} c_i^2.$$

In 1748 Euler knew solutions to this problem for $n = 2$, 3, 4. However, A. Hurwitz in 1898 shown that for $n > 4$ the solution exists only for $n = 8$. The correspondent solution was found before Hurwitz

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by the outstanding Italian geometrician Francesco Brioschi (1824–1897) [3], who used special complex projective coordinates (16-spinors Ψ) to study the geometry of the 8-dimensional space.

In particular, Brioschi proved the remarkable identity satisfied by any 8-spinor ψ (semispinor):

$$j^\mu j_\mu - \tilde{j}^\mu \tilde{j}_\mu = s^2 + p^2 + \vec{v}^2 + \vec{a}^2, \tag{1}$$

where the standard bilinear spinor quantities are used:

$$s = \bar{\psi}\psi, \quad p = \imath\bar{\psi}\gamma_5\,\psi, \quad \vec{v} = \bar{\psi}\vec{\lambda}\,\psi,$$

$$\vec{a} = \imath\bar{\psi}\gamma_5\vec{\lambda}\,\psi, \quad j_\mu = \bar{\psi}\gamma_\mu\psi, \quad \tilde{j}_\mu = \bar{\psi}\gamma_\mu\gamma_5\,\psi,$$

with Dirac matrices γ_μ, γ_5 and Pauli isotopic ones $\vec{\lambda}$ being included.

The structure of the invariant (1) is discussed in the paper [8] and also in the manual [9].

Now it should be remarked that in view of the special structure of the invariant (1) the principle of spontaneous symmetry breaking comes into play. In fact, suppose that in our Ψ-field model the Lagrangian comtains the Higgs potential, which is proportional to some positive degree of the quantity (1). Therefore, fixing the vacuum value of (1), one finds the field manifold

$$s^2 + p^2 + \vec{v}^2 + \vec{a}^2 = \text{const},$$

which is homeomorphic to S^7. On the other hand, the following inclusions take place:

$$S^7 \supset S^3 \supset S^2. \tag{2}$$

In view of (2) and due to the nontriviality of the homotopy groups

$$\pi_3(S^3) = \pi_3(S^2) = \mathbf{Z}$$

one can expect the existence of topological solitons endowered with the winding number $\deg(\mathrm{S}^3 \to \mathrm{S}^3)$ or the Hopf index Q_H, which can be interpreted as the baryon or the lepton numbers, respectively. The corresponding models proved their effectiveness in nuclear physics (Skyrme model [4]) and in the theory of lepton interactions (Faddeev model [5]).

2. Wiener interpretation of quantum mechanics

Wiener found a special representation of the wave function considered as an element of the random Hilbert space with the Gaussian dispersion [6]. If one follows the Einstein's idea of particles-solitons, it means that the wave function behaves as a sum of complex soliton configurations with random phases. Let us recall that in accordance with the central limiting theorem [7] this sum is equivalent to a Gaussian stochastic variable.

As an illustration of this effect let us consider the classical T. Young's n-slit diffraction experiment, the photons being replaced with the Brioschi solitons $\phi(t,\vec{x})$. Introducing the Lagrangian density $\mathcal{L}(\phi,\,\partial_\mu\phi)$ for our field model and calculating the canonical momentum

$$\pi(t,\vec{x}) = \frac{\partial\mathcal{L}}{\partial(\partial_t\phi)},$$

one can construct the auxiliary complex function

$$\varphi(t,\vec{x}) = 2^{-1/2}\left(\nu\varphi + \imath\pi/\nu\right), \tag{3}$$

where the number ν can be found from the normalization:

$$\hbar = \int d^3x\,|\varphi|^2,$$

with $\hbar$ being the Planck constant. It is worth while to underline that the quantity (3) plays the role of the "coherent state soliton"and simplifies the calculation of physical observables.

Let us denote by $N >> 1$ the number of trials for scattering the Brioschi soliton on the n-slit screen during the Young's experiment. Denoting $\varphi_j(t, \vec{x})$ the soliton profile for the j-th trial, one defines the wave function in the stochastic representation or the probability amplitude for measuring the coordinates of the soliton's center as follows [8]:

$$\psi(t, \vec{x}) = (\hbar N)^{-1/2} \sum_{j=1}^{N} \varphi_j(t, \vec{x}). \tag{4}$$

Considering φ_j and $\varphi_{j'}$ for $j \neq j'$ as independent random variables, one concludes that the wave function (4) is equivalent to the Wiener's one due to the central limiting theorem. As was shown in [8], the definition (4) of the wave function satisfies the correspondence condition with the Born's rule of quantum mechanics for calculating mean values of physical observables as bilinear functionals with respect to ψ.

Conclusion

The main goal of the present work appears to search for physically clear arguments in favour of using soliton configurations for describing the internal structure of elementary particles. Precisely this idea served the foundation of the tremendous Einstein program of geometrizing physics. The important contribution to realizing this program was made by the outstanding Italian geometrician F. Brioschi, who used complex projective coordinates to study the geometry of the 8-dimensional space. At last, the brilliant mathematician N. Wiener proved the essential role of the internal particle structure for quantum description of the world.

Acknowledgement:

This work being recommended to publishing by the Organizational Committee, the author expresses his sincere gratitude to the organizers of the XXIV International Meeting "Physical Interpretation of Relativity Theory PIRT-2025"at the Bauman Moscow State Technical University and especially to professor K.A. Postnov [Sternberg Astronomical Institute, Moscow State University] for fruitful discussions on relativistic quantum mechanics.

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Просьба ссылаться на эту статью следующим образом:

Рыбаков Ю. П. Подход Винера–Бриоски к релятивистской квантовой механике. *Пространство, время и фундаментальные взаимодействия.* 2025. № 4. С. 106—109.

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Please cite this article in English as:

Rybakov Yu. P. Wiener–Brioschi approach to relativistic quantum mechanics. *Space, Time and Fundamental Interactions*, 2025, no. 4, pp. 106–109.