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СПИНОРНОЕ ПОЛЕ В ЛОКАЛЬНО ВРАЩАТЕЛЬНО-СИММЕТРИЧНОЙ КОСМОЛОГИИ ТИПА БИАНКИ-I С ГЕОМЕТРИЕЙ ЛИРЫ

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В данной работе мы рассматриваем роль нелинейного спинорного поля в эволюции Вселенной в рамках локально вращательно-симметричной космологической модели типа Бианки I (LRSBI) с геометрией Лиры. В предыдущих исследованиях нелинейное спинорное поле изучалось в различных анизотропных и изотропных космологических моделях, что показало, что наличие нетривиальных, недиагональных компонент тензора энергии-импульса спинорного поля накладывает существенные ограничения как на геометрию пространства-времени, так и на само спинорное поле. В нашей текущей работе мы обнаруживаем, что, хотя эти ограничения остаются в силе, введение геометрии Лиры существенно влияет на эволюцию Вселенной. Это влияние обусловлено тем, что инварианты спинорного поля зависят от параметра геометрии Лиры.

Ключевые слова: спинорное поле, анизотропные космологические модели, тензор энергии-импульса, геометрия Лиры.

SPINOR FIELD IN LOCALLY ROTATIONALLY SYMMETRIC BIANCHI TYPE-I COSMOLOGY WITH LYRA'S GEOMETRY

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In this study, we examine the role of a nonlinear spinor field in the evolution of the Universe within the framework of a Locally Rotationally Symmetric Bianchi type-I (LRSBI) cosmological model with Lyra's geometry. Previous research has explored the nonlinear spinor field in various anisotropic and isotropic cosmological models, revealing that the presence of nontrivial, non-diagonal components of the spinor field's energy-momentum tensor imposes severe restrictions on both the space-time geometry and the spinor field itself. In our current study, we find that while these restrictions still apply, the introduction of Lyra's geometry significantly influences the evolution of the Universe. This influence arises from the fact that the invariants of the spinor field are dependent on the Lyra geometry parameter.

Keywords: spinor field, anisotropic cosmological models, energy-momentum tensor, Lyra's geometry.

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Introduction

The Standard Model of Cosmology (SMC), also known as the Λ-CDM model (Λ-Cold Dark Matter), rests on three fundamental assumptions:

(a) the validity of General Relativity on cosmological scales;

(b) the correctness of the Standard Model of particle physics at small (quantum) scales; and

(c) the cosmological principle, which posits that the Universe is spatially homogeneous, isotropic, and infinite on large scales.

According to this model, the Universe originated from a Big Bang, emerging from a state of pure energy. The present-day energy composition of the Universe is estimated to be approximately 5% ordinary (baryonic) matter, 27% dark matter, and 68% dark energy.

Despite its simplicity, the Λ-CDM model successfully explains a wide range of cosmological observations, including Type Ia supernovae, cosmic microwave background radiation (CMBR) anisotropies, large-scale structure formation, gravitational lensing, and baryon acoustic oscillations. However, it faces theoretical challenges, notably severe fine-tuning problems related to the vacuum energy (cosmological constant) scale. These shortcomings motivate the exploration of alternative cosmological models. In such alternatives, researchers often seek to modify Einstein's field equations by introducing additional terms in the gravitational Lagrangian beyond the Ricci scalar or by considering non-Riemannian geometries. Some approaches also involve exotic matter or field sources.

Shortly after Einstein proposed his famous theory of gravity, Weyl in an attempt to unify gravitation and electromagnetic field, introduced a generalization of Riemannian Geometry [1]. Weyl theory was not taken seriously as it contradicted some well-known observational result. In 1951 Lyra proposed a modification of Riemannian geometry which bears a close resemblance of Weyl geometry [2]. But unlike Weyl geometry, in Lyra's geometry the connection is metric preserving as in Riemannian geometry. In doing so he introduced a gauge function into the structureless manifold. This theory was further developed by Sen [3], Halford [4], Sen and Dunn [5], Sen and Vanstone [6] and many others. Recently Lyra's geometry is being used extensively in cosmology [7–11].

In a number of papers [12–14] it was shown that spinor field is very sensitive to the gravitational one. In most cases there exist nontrivial non-diagonal components of energy-momentum tensor (EMT) which leads to the different types of restrictions both on the geometry of space-time and the spinor field itself. The aim for considering Lyra's geometry is to clarify whether it can remove or weaken the restrictions those occur in usual cases.

1. Basic equations

In this section we give a short description of Lyra's geometry, spinor field and obtain the gravitational field equations for a Bianchi type-I cosmological model.

1.1. Lyra's geometry

Lyra suggested a modification of Riemannian geometry which is also a modification of Weyl geometry. The metrical concept of gauge in Weyl geometry was modified by a structureless gauge function. According to Lyra's geometry the displacement vector from a point $P(x^\mu)$ to a neighbouring point $P'(x^\mu + dx^\mu)$ is defined by $\xi^\mu = x^0 dx^\mu$, where x^0 is a nonzero analytical function of coordinates and fixes the gauge of the system. Together with coordinate system x^μ, x^0 form a so-called reference system (x^0, x^μ). The transformation to a new reference system is given by

$$x^\mu = x^\mu(\bar{x}^1, \cdots, \bar{x}^n), \quad x^0 = x^0(\bar{x}^1, \cdots, \bar{x}^n, \bar{x}^0), \tag{1}$$

where $\partial x^0/\partial \bar{x}^0 \neq 0$ and $\det \partial x^\mu/\partial \bar{x}^\nu \neq 0$. Under the transformation (1) a contravariant vector ξ^μ is transformed according to

$$\bar{\xi}^\mu = \lambda A^\mu_\nu \xi^\nu, \quad A^\mu_\nu = \frac{\partial \bar{x}^\mu}{\partial x^\nu}, \quad \lambda = \frac{\bar{x}^0}{x^0}, \tag{2}$$

with λ being the gauge factor of transformation.

In any general reference system $(x^0,\, x^\mu)$ the infinitesimal parallel transfer of a vector from $P(x^\mu)$ to $P'(x^\mu + dx^\mu)$ can be expressed as

$$\delta\xi^\mu = -\tilde{\Gamma}^\mu_{\alpha\beta}\xi^\alpha x^0 dx^\beta, \quad \tilde{\Gamma}^\mu_{\alpha\beta} = \Gamma^\mu_{\alpha\beta} - \frac{1}{2}\delta^\mu_\alpha\phi_\beta, \quad \phi_\alpha = -\frac{\partial(\log\lambda^2)}{\partial x^\alpha}. \tag{3}$$

It should be noted that $\Gamma^\mu_{\alpha\beta} = \Gamma^\mu_{\beta\alpha}$, though $\tilde{\Gamma}^\mu_{\alpha\beta} \neq \tilde{\Gamma}^\mu_{\beta\alpha}$.

Since the displacement vector between two neighbouring points $P(x^\mu)$ and $P'(x^\mu + dx^\mu)$ in this case is define by $\xi^\mu = x^0 dx^\mu$, the interval between them is given by the invariant

$$ds^2 = g_{\mu\nu} x^0 dx^\mu x^0 dx^\nu, \tag{4}$$

where $g_{\mu\nu}$ is the symmetric tensor of second rank. The parallel transport of length in Lyra geometry is integrable, i.e., $\delta(g_{\mu\nu}\xi^\mu\xi^\nu) = 0$ and the connection $\Gamma^\alpha_{\mu\nu}$ in (3) takes form

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{x^0}\{^{\,\alpha}_{\mu\nu}\} + \frac{1}{2}\left(\delta^\alpha_\mu\phi_\nu + \delta^\alpha_\nu\phi_\mu - g_{\mu\nu}\phi^\alpha\right), \tag{5}$$

which is similar to that of Weyl geometry except the multiplier $1/x^0$. Here $\{^{\,\mu}_{\alpha\beta}\}$ is the Levi-Civita connection. Note that in Lyra geometry the derivative $\partial/\partial x^\mu$ is substitute by $\partial/(x^0\partial x^\mu) = (1/x^0)\partial/\partial x^\mu$. As one sees, $\Gamma^\alpha_{\mu\nu} = \Gamma^\alpha_{\nu\mu}$. On account of (5) from (3) we find

$$\tilde{\Gamma}^\alpha_{\mu\nu} = \frac{1}{x^0}\{^{\,\alpha}_{\mu\nu}\} + \frac{1}{2}\left(\delta^\alpha_\nu\phi_\mu - g_{\mu\nu}\phi^\alpha\right). \tag{6}$$

The parallel transfer, hence the equation of motion

$$\frac{1}{x^0}\frac{\partial\xi^\alpha}{\partial\beta} + \tilde{\Gamma}^\alpha_{\nu\beta}\xi^\nu = 0, \tag{7}$$

can be integrated if the components of the tensor

$$K^\lambda_{\mu\alpha\beta} = \frac{1}{(x^0)^2}\left[\frac{\partial(x^0\tilde{\Gamma}^\lambda_{\mu\beta})}{\partial x^\alpha} - \frac{\partial(x^0\tilde{\Gamma}^\lambda_{\mu\alpha})}{\partial x^\beta} + x^0\tilde{\Gamma}^\lambda_{\rho\alpha}x^0\tilde{\Gamma}^\rho_{\mu\beta} - x^0\tilde{\Gamma}^\lambda_{\rho\beta}x^0\tilde{\Gamma}^\rho_{\mu\alpha}\right], \tag{8}$$

vanish [3]. Einstein's field equation in Lyra's geometry in normal gauge ($x^0 = 1$) was found by Sen [3] and can be written as

$$G^\nu_\mu + \frac{3}{2}\phi_\mu\phi^\nu - \frac{3}{4}\delta^\nu_\mu\phi_\alpha\phi^\alpha = \kappa T^\nu_\mu, \quad G^\nu_\mu = R^\nu_\mu - \frac{1}{2}\delta^\nu_\mu R. \tag{9}$$

where ϕ_μ is the displacement vector.

1.2. Spinor field

We consider the spinor field Lagrangian given by [12]

$$L_{\rm sp} = \frac{\imath}{2}\left[\bar{\psi}\gamma^\mu\nabla_\mu\psi - \nabla_\mu\bar{\psi}\gamma^\mu\psi\right] - m_{\rm sp}\bar{\psi}\psi - \lambda F, \tag{10}$$

where the nonlinear term F describes the self-interaction of a spinor field and can be presented as some arbitrary functions of invariants K that takes one of the following values $\{I,\, J,\, I+J,\, I-J\}$ generated from the real bilinear forms of a spinor field. We also consider the case $\psi = \psi(t)$ so that $I = S^2 = (\bar{\psi}\psi)^2$, & $J = P^2 = (\imath\bar{\psi}\gamma^5\psi)^2$. Here λ is the self-coupling constant.

The spinor field equations take the form

$$\imath\gamma^{\mu}\nabla_{\mu}\psi - m_{\rm sp}\psi - \mathcal{D}\psi - \imath\mathcal{G}\gamma^5\psi = 0, \tag{11}$$

$$\imath\nabla_{\mu}\bar{\psi}\gamma^{\mu} + m_{\rm sp}\bar{\psi} + \mathcal{D}\bar{\psi} + \imath\mathcal{G}\bar{\psi}\gamma^5 = 0, \tag{12}$$

where we denote $\mathcal{D} = 2SF_K K_I$ and $\mathcal{G} = 2PF_K K_J$ with $F_K = dF/dK$, $K_I = dK/dI$ and $K_J = dK/dJ$. In the Lagrangian (10) and spinor field equations (11) and (12) ∇_μ is the covariant covariant derivative of the spinor field so that $\nabla_\mu\psi = \partial_\mu - \Omega_\mu\psi$ and $\nabla_\mu\bar{\psi} = \partial\bar{\psi} + \bar{\psi}\Omega_\mu$. In Lyra's geometry we should substitute $\Gamma^\rho_{\mu\nu}$ with $\tilde{\Gamma}^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} - (1/2)\delta^\rho_\mu\phi_\nu = \frac{1}{x^0}\{^{\alpha}_{\mu\nu}\} + \frac{1}{2}\left(\delta^\rho_\nu\phi_\mu - g_{\mu\nu}\phi^\rho\right)$. Thus in this case we have

$$\begin{aligned}\tilde{\Omega}_\mu &= \frac{1}{4}\bar{\gamma}_a\gamma^\nu\partial_\mu e^{(a)}_\nu - \frac{1}{4}\gamma_\rho\gamma^\nu\tilde{\Gamma}^\rho_{\mu\nu}\\ &= \frac{1}{4}\bar{\gamma}_a\gamma^\nu\partial_\mu e^{(a)}_\nu - \frac{1}{4}\gamma_\rho\gamma^\nu\{^{\rho}_{\mu\nu}\} + \frac{1}{8}\left(\gamma_\rho\gamma^\rho\phi_\mu - \gamma_\rho\gamma_\mu\phi^\rho\right).\end{aligned} \tag{13}$$

The energy momentum tensor of the spinor field is given by

$$\begin{aligned}T_\mu{}^\rho &= \frac{\imath g^{\rho\nu}}{4}\left(\bar{\psi}\gamma_\mu\nabla_\nu\psi + \bar{\psi}\gamma_\nu\nabla_\mu\psi - \nabla_\mu\bar{\psi}\gamma_\nu\psi - \nabla_\nu\bar{\psi}\gamma_\mu\psi\right) - \delta^\rho_\mu L_{\rm sp}\\ &= \frac{\imath}{4}g^{\rho\nu}\left(\bar{\psi}\gamma_\mu\partial_\nu\psi + \bar{\psi}\gamma_\nu\partial_\mu\psi - \partial_\mu\bar{\psi}\gamma_\nu\psi - \partial_\nu\bar{\psi}\gamma_\mu\psi\right)\\ &- \frac{\imath}{4}g^{\rho\nu}\bar{\psi}\left(\gamma_\mu\tilde{\Omega}_\nu + \tilde{\Omega}_\nu\gamma_\mu + \gamma_\nu\tilde{\Omega}_\mu + \tilde{\Omega}_\mu\gamma_\nu\right)\psi\\ &- \delta^\rho_\mu\lambda\left(2KF_K - F(K)\right).\end{aligned} \tag{14}$$

On account of spinor field equations (11) and (12) the spinor field Lagrangian takes the form $L_{\rm sp} = 2KF_K - F(K)$. Thanks to spinor field equations the conservation of energy holds, i.e.,

$$T^\mu_{\nu;\mu} = 0. \tag{15}$$

As we see later, this very fact will help us to define the parameter of Lyra's geometry in terms of volume scale.

1.3. LRS Bianchi type-I model

The locally rotationally symmetric Bianchi type-I (LRS-BI) space-time we take in the form

$$ds^2 = dt^2 - a_1^2\left(dx_1^2 + dx_2^2\right) - a_3^2 dx_3^2, \tag{16}$$

with $a_1,\ a_3$ being the functions of time only. Following Beesham [7] we consider the gauge function as follows:

$$\phi_\mu = \{\beta(t),\, 0,\, 0,\, 0\}, \tag{17}$$

The spinor affine connection in this case has the form:

$$\Omega_0 = -\frac{3}{8}\beta, \tag{18a}$$

$$\Omega_1 = \frac{1}{2}\left(\dot{a}_1 + \frac{\beta a_1}{4}\right)\bar{\gamma}^1\bar{\gamma}^0, \tag{18b}$$

$$\Omega_2 = \frac{1}{2}\left(\dot{a}_1 + \frac{\beta a_1}{4}\right)\bar{\gamma}^2\bar{\gamma}^0, \tag{18c}$$

$$\Omega_3 = \frac{1}{2}\left(\dot{a}_3 + \frac{\beta a_3}{4}\right)\bar{\gamma}^3\bar{\gamma}^0. \tag{18d}$$

Thus we see that the introduction of Lyra geometry bring changes in Ω_μ. The spinor field equations now take the form

$$\frac{1}{x^0}\dot{\psi} + \frac{\dot{V}}{2V}\psi + \frac{3}{4}\beta\psi + \imath\left(m_{\rm sp} + \mathcal{D}\right)\bar{\gamma}^0\psi - \mathcal{G}\bar{\gamma}^0\bar{\gamma}^5\psi = 0, \tag{19a}$$

$$\frac{1}{x^0}\dot{\bar{\psi}} + \frac{\dot{V}}{2V}\bar{\psi} - \imath\left(m_{\rm sp} + \mathcal{D}\right)\bar{\psi}\bar{\gamma}^0 - \mathcal{G}\bar{\psi}\bar{\gamma}^0\bar{\gamma}^5 = 0, \tag{19b}$$

with V being the volume scale:

$$V = a_1^2 a_3. \tag{20}$$

Note that in Lyra's geometry the differential operator $\partial/\partial x^\mu$ is substituted by $(1/x^0)\partial/\partial x^\mu$. But in natural gauge with $x^0 = 1$ we omit it in our further calculations.

For the invariants we find

$$\dot{S}_0 + \frac{3}{4}\beta S_0 + 2\mathcal{G}A_0^0 = 0, \tag{21a}$$

$$\dot{P}_0 + \frac{3}{4}\beta P_0 + 2\left(m_{\rm sp} + \mathcal{D}\right) A_0^0 = 0, \tag{21b}$$

$$\dot{A}_0^0 + \frac{3}{4}\beta A_0^0 + 2\left(m_{\rm sp} + \mathcal{D}\right) P_0 - 2\mathcal{G}S_0 = 0, \tag{21c}$$

with the solutions

$$S_0^2 - P_0^2 + {A_0^0}^2 = C_0 \exp[-(3/2)\int \beta(t)dt], \tag{22}$$

with C_0 being some constant of integration. Here $S_0 = SV$, $P_0 = PV$ and $A_0^0 = A^0 V$.

On account of (18) we find the following diagonal components of the energy momentum tensor of the spinor field

$$T_0^0 = m_{\rm sp} S + \lambda F = \varepsilon, \quad T_1^1 = T_2^2 = T_3^3 = \lambda\left(F - 2KF_K\right) = -p. \tag{23}$$

Note that they are identical to those found in [13,14] where Lyra's geometry was not exploited. The reason is the additional terms occurred in this case canceled each other. Same thing happens for non-diagonal components as well. The nontrivial non-diagonal components of EMT in this case read [13,14]:

$$T_1^3 = \frac{1}{4}\frac{a_1}{a_3}\left(\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1}\right) A^2, \qquad T_3^2 = \frac{1}{4}\frac{a_3}{a_2}\left(\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3}\right) A^1. \tag{24}$$

As we will find later, these non-diagonal components of EMT will characterize the geometry of space-time or spinor field.

1.4. Einstein equation and its solution

Let us recall that Einstein tensor corresponding to LRS-BI space-time (16) possesses only diagonal components. The diagonal components of Einstein's equations are [13,14]:

$$\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1} - \frac{3}{2}\beta^2 = \kappa\lambda\left(F - 2KF_K\right), \tag{25a}$$

$$2\frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1^2}{a_1^2} - \frac{3}{2}\beta^2 = \kappa\lambda\left(F - 2KF_K\right), \tag{25b}$$

$$\frac{\dot{a}_1^2}{a_1^2} + 2\frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1} + \frac{3}{2}\beta^2 = \kappa\left(m_{\rm sp} S + \lambda F\right). \tag{25c}$$

Thanks to the fact that $T_1^1 = T_2^2 = T_3^3$ from (25a) - (25b) for metric functions a_i one finds

$$a_i = X_i V^{1/3} \exp[Y_i \int (1/V) dt], \quad X_1^2 X_3 = 1, \quad 2Y_1 + Y_3 = 0, \quad i = 1,3. \tag{26}$$

Here X_i and Y_i are the constants of integration. Thus we see that the metric functions are given in terms of V. Hence we have to find the volume scale as well. From (25) for volume scale we obtain

$$\ddot{V} = \kappa\left[m_{\rm sp} S + 6\lambda\left(F - KF_K\right)\right] V. \tag{27}$$

As one sees, the equation (27) does not contain β, hence identical to the case without Lyra's geometry. From (21) it can be shown that

$$S = \frac{C_0}{V}\exp\left[-\frac{3}{4}\int\beta(t)dt\right], \quad K = \frac{C_0^2}{V}\exp\left[-\frac{3}{2}\int\beta(t)dt\right], \quad C_0 = \text{const.} \tag{28}$$

Note that for $K = \{J,\, I \pm J\}$ relations (28) holds for massless spinor field only, whereas for $K = I$ it is true for both massive and massless spinor field. Thus we see that the equation (27) implicitly depends on β that defines Lyra geometry.

Taking into account that the in case of spinor field $T^{\nu}_{\mu;\nu} = 0$ on accout on Bianchi identity $G^{\nu}_{\mu;\nu} = 0$ from (9) we find

$$\left(\frac{3}{2}\phi_\mu\phi^\nu - \frac{3}{4}\delta^\nu_\mu\phi_\alpha\phi^\alpha\right)_{;\nu} = V\dot\beta + \beta\dot V = 0, \tag{29}$$

with the solution

$$\beta = \beta_0/V, \quad \beta_0 = \text{const.} \tag{30}$$

The R.H.S. of equation (27) depends on V only as now we have

$$S = \frac{C_0}{V}\exp\left[-\frac{3\beta_0}{4}\int\frac{dt}{V(t)}\right], \quad K = \frac{C_0^2}{V^2}\exp\left[-\frac{3\beta_0}{2}\int\frac{dt}{V(t)}\right], \quad C_0 = \text{const.} \tag{31}$$

In what follows, stead of (27) we solve the system (25), numerically.

1.5. Numerical solutions

In this section we solve the system (25). Introducing directional Hubble parameters we rewrite the equations (25a) and (25b) as:

$$\dot a_1 = H_1 a_1, \tag{32a}$$

$$\dot a_3 = H_3 a_3, \tag{32b}$$

$$\dot H_1 = \frac{\kappa}{2}\lambda\,(F - 2KF_K) + \frac{3}{4}\frac{\beta_0^2}{a_1^2a_2^2a_3^2} - \frac{3}{2}H_1^2, \tag{32c}$$

$$\dot H_3 = \frac{\kappa}{2}\lambda\,(F - 2KF_K) + \frac{3}{4}\frac{\beta_0^2}{a_1^2a_2^2a_3^2} + \frac{1}{2}H_1^2 - H_3^2 - H_3H_1. \tag{32d}$$

whereas the equation (25c) we exploit to find initial value.

To solve the equation (27) numerically we have to give the concrete form of spinor field nonlinearity. It was found earlier that the spinor field nonlinearity can simulate different types of source field such as quintessence, Chaplygin gas, modified quintessence, modified Chaplygin gas etc. [14]:

$$F(K) = \lambda_1 K^{(1+W)/2} - m_{\rm sp}S, \quad W = \text{const.} - \text{quintessence}, \tag{33a}$$

$$F(K) = \left(A + \lambda_1 K^{(1+\alpha)/2}\right)^{1/(1+\alpha)}, \ A > 0, \quad 0 \le \alpha \le 1, \ - \text{Chaplygin gas}, \tag{33b}$$

$$F(K) = \lambda_1 K^{(1+W)/2} + \frac{W}{\lambda(1+W)}\varepsilon_{\rm cr}, - \text{modified quintessence}, \tag{33c}$$

$$F(K) = \left[\frac{B}{1+W} + \lambda_1 K^{(1+\alpha)(1+W)/2}\right]^{1/(1+\alpha)}, \quad - \text{modified Chaplygin gas}, \tag{33d}$$

where λ_1 is the integration constant.

It should be emphasized that only in case of quintessence the spinor field can me massive, whereas in other cases it is massless. Moreover, the massive term in (33a) cancels the massive term in Lagrangian

(10). Hence we can dully set $m_{\rm sp} = 0$ in our further calculations. Now the equation (25c) in view of (32a), (32b) and (31) takes the form

$$H_1^2 + 2H_3H_1 = \kappa F - \frac{3\beta_0^2}{2V^2}. \tag{34}$$

Given the initial values of a_1, a_2, a_3, H_2, H_3 one can find

$$H_3 = \frac{\kappa F - 3\beta_0^2/(2a_1^2a_2^2a_3^2) - H_1^2}{2H_1}. \tag{35}$$

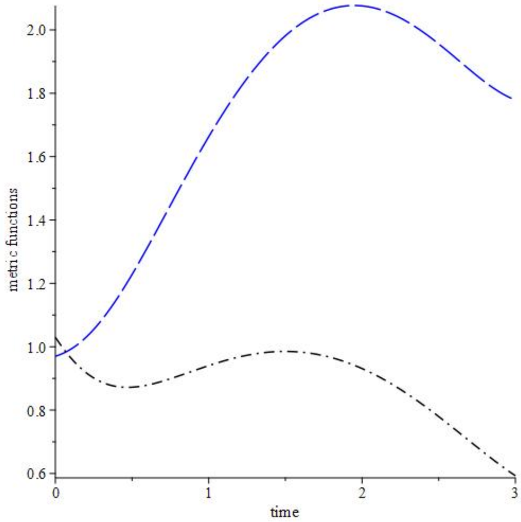

Fig. 1. Evolution of metric functions $a_1(t)$ (blue long dash) and $a_3(t)$ (black dash-dot) with Lyra geometry when spinor field nonlinearity simulates modified Chapligin gas with positive W.

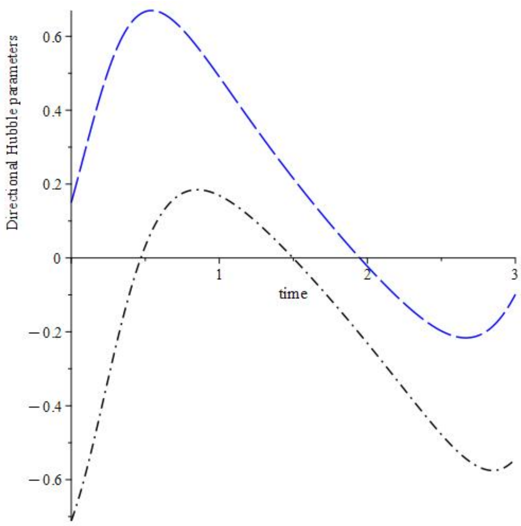

Fig. 2. Evolution of directional Hubble parameters $H_1(t)$ (blue long dash) and $H_3(t)$ (black dash-dot) with Lyra geometry when spinor field nonlinearity simulates modified Chapligin gas with positive W.

In what follows we solve the equation (27) numerically. Our aim is to clarify whether the introduction of Lyra's geometry bring any essential changes in the solution. For simplicity we set $m_{\rm sp} = 0$, $\kappa = 1$, $\lambda = 1$, $\lambda_1 = 1$. Here we consider modified Chaplygin gas with $B = 1$, $W = 1/2$ and $\alpha = 1/3$. For initial values we set $a_1(0) = 0.97$, $a_3(0) = 1.03$ and $H_1(0) = 0.15$. Initial value for $H_2(0)$ is calculated form (35) that in this case gives $H_3(0) = -0.71$. In Figures 1 and 2 we display the evolution of metric functions $a_i(t)$ and directional Hubble parameters $H_i(t)$ for modified quintessence. In the figures blue long das, red dash-dot and black solid line stand components of metric functions and directional Hubble parameters along $X-$, $Y-$ and $Z-$ axes, respectively.

In Figues 3 and 4 we display the analogical graphs for $W = -1/2$. In this case from (35) we find $H_3(0) = 1.89$

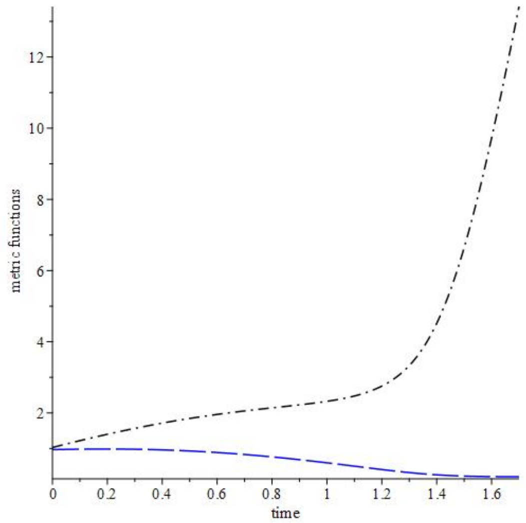

Fig. 3. Evolution of metric functions $a_1(t)$ (blue long dash) and $3_2(t)$ (black dash-dot) with Lyra geometry when spinor field nonlinearity simulates modified Chapligin gas with negative W.

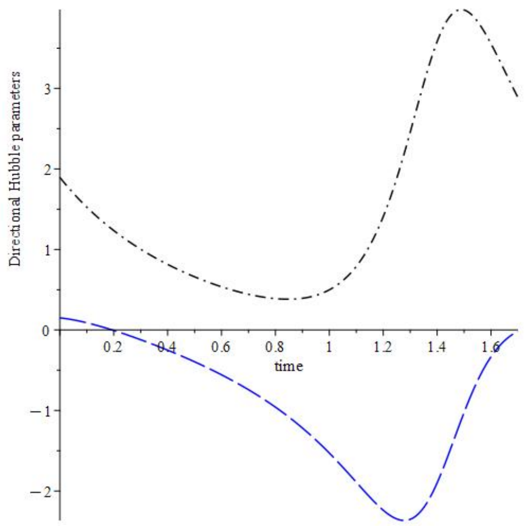

Fig. 4. Evolution of directional Hubble parameters $H_1(t)$ (blue long dash) and $H_3(t)$ (black dash-dot) with Lyra geometry when spinor field nonlinearity simulates modified Chapligin gas with negative W.

Concluding remarks

Within the framework of a LRS Bianchi type-I cosmological model, we investigate the role of Lyra's geometry in the evolution of the Universe when it is filled with dark energy modeled by a spinor field. Our analysis reveals that the corresponding Einstein equations retain the same form as in the absence of Lyra's geometry. However, the dependence of spinor field invariants on the Lyra geometry parameter influences the final results.

As in the standard case, the presence of nontrivial, non-diagonal components in the energy-momentum tensor (EMT) of the spinor field leads to three possible scenarios. In the case of a general Bianchi type-I model, the spinor field becomes massless and linear, whereas in case of LRSBI model spinor field nonlinearity does not suffer. The relevant equations are solved numerically, with the solutions presented graphically.

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